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A new analytical model of ultimate water cut for light oil reservoirs with bottom-water

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Abstract

Ultimate water cut (WCult) defines well's maximum water production for uncontained oil pay with bottom-water. The WCult is important to determine if the reservoir development is economical. Since presently-used WCult formula derives from simplifying assumption ignoring the effect of non-radial inflow, the formula needs to be redefined. A new analytical formula of WCult is developed by considering the inflow of oil and water into separate completions at the top of oil-zone and aquifer respectively. Then the formula is verified using the design of 46 simulated experiments representing wide variety of reservoir-bottomwater systems. It was found out that the for light-oil reservoirs, the presently-used theoretical formula may significantly diverge from the proposed formula which closely matches the simulated data and is more physics driven. Hence the proposed formula should be preferred. However, for the viscous oil reservoirs, the presently used formula conforms to the proposed formula, which is also proved mathematically.

Keywords: Ultimate water-cut, light oil, bottom-water reservoir, water coning, partial penetration

Introduction

Ultimate water-cut is a maximum stabilized water cut in an oil-pay affected by water coning. The scenario is physically modeled by setting a balanced-oil-rate (BOR) boundary of the well's drainage area by replacing the produced oil at the the drainage boundary. After the water break-through time, there is an initial rapid increase of water-cut representing the water cone development stage, followed by the stabilization period until the WC value becomes constant, WCult.

Kuo and Desbrisay[1] introduced the concept and formula of ultimate water-cut[2]:

$$WCult = \frac{Mh_w}{Mh_w + h_o} \tag{1}$$

Shirman and Wojtanowicz[3] showed that WCult in DWS wells is always lower than that in conventional wells. Their experimental results revealed that it is possible to completely reduce WCult to zero at high drainage rates. Other authors[3–5] showed the dependence of ultimate water-cut on production rate. For production rates slightly higher than critical rates (maximum possible production rate without water breakthrough), water-cut would stabilize at value lower than that in Eq. (1). After conducting laboratory experiments, Shirman and Wojtanowicz[3] found out that the water-cut stabilization value may not predict the Kuo and Desbrisay[1] model at low production rate. They modified Eq. (1) by including the effect of production-rate as,

$$WCult = (1 - \tfrac{q_{cr}}{Q})\frac{Mh_w}{Mh_w + h_o} \tag{2}$$

Both Eqs. (1) and (2) assume the radial flow in the oil-zone and aquifer having a BOR boundary depicted in Figure 1, and there by ignores any nonradial distorted inflows (in oil-zone and aquifer) to a partially penetrating well. Prasun and Wojtanowciz[6,7] attempted to include the effect of partial-penetration in the closed-boundary reservoirs. However, they found that the new modified WCult formula reduces back to the original formula (Eq. (1)); thus disapproving any effect of partial-penetration on ultimate water-cut in these reservoirs. Apparently, they verified the effect of partial penetration by comparing the formula with the results from the wide variety of NFRs. However, they failed to understand that the generalized consideration of all attributes of reservoirs while verification, may conceal the partial-penetration effects for certain types of reservoirs. So, this study derives a new model of ultimate water-cut for the BOR systems considering the non-radial inflow to a partial-penetrating well, and then verifies it with particular types of reservoirs classified as light oil and viscous oil reservoirs. A good match for the particular reservoir, would justify the relevance of the partial penetration effects for this reservoir.

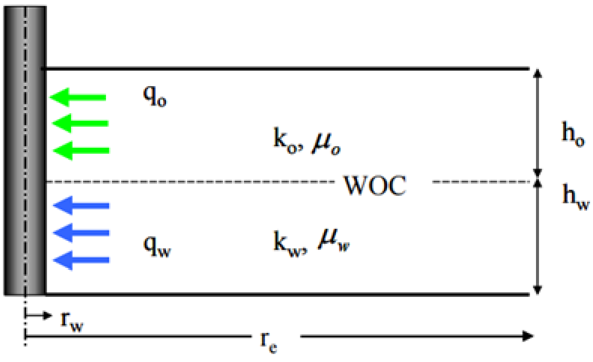

Figure 1 Oil and water horizontal flow in their respective zones.

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Modified analytical formula of ultimate water-cut

In derivation of a new ultimate water-cut model for a partially penetrating well in BOR system, we consider the following assumptions:

There is a piston-like displacement of oil by coned water flowing into the well. So, the rising water cone development covers larger area of oil completion before final stabilization. Eventually, the ratio of well completion producing oil and water becomes equal to the ratio of oil and water zone thickness, when ultimate water-cut is reached.[3]

In a piston-like displacement, there is almost no mixing between the flow regions of oil and water. Assumption 1 follows that the partially penetrating oil completion region (producing only oil) is at the top of oil-zone, whereas, for simplicity, we assume the partially penetrating water completion region (producing only water) is displaced from the oil-zone to the top of aquifer as shown in Figure 2. This assumption ignores the additional skin due to the water inflow from aquifer to the completion in oil-zone.

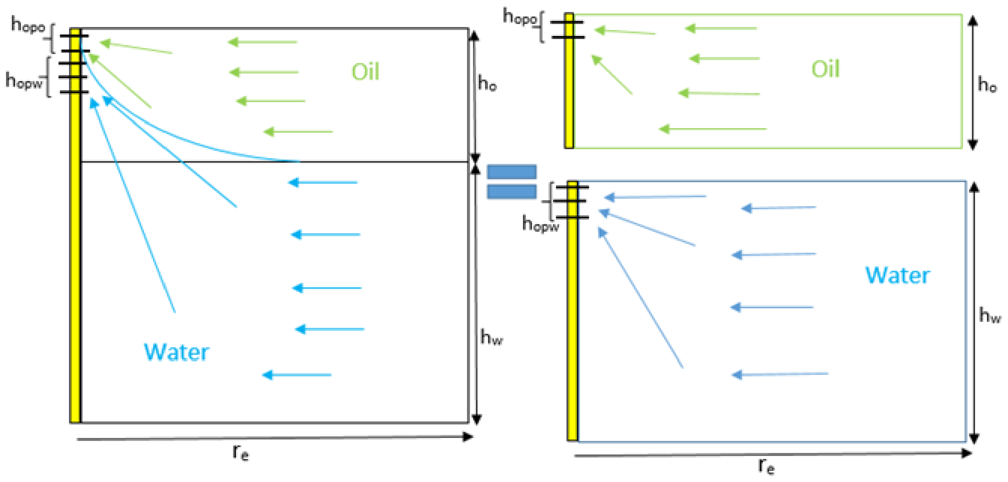

Figure 2 Equivalence of oil and water inflow schematic between combined and separate systems.

Darcy-law flow-rate equations of oil (q_o) and water (q_W) well-inflow (into their respective completions) during ultimate water-cut stage, at surface conditions, can be given by (Appendix A),

$$q_o = \frac{2\pi k_h k_{ro} h_o (p_e - p_w)}{\mu_o B_o (\ln {}^{r_e}\!/_{r_w} + s_o)} \tag{3}$$

$$q_w = \frac{2\pi k_h k_{rw} hw (p_e - p_w)}{\mu_w B_w (\ln {}^{r_e}\!/_{r_w} + s_w)} \tag{4}$$

where, r_e is the radial size of reservoir, ft; S_o is the skin factor due to oil-inflow defined by Eq. (A-4); S_w is the skin factor due to water-inflow defined by Eq. (A-7); r_w is the well radius, ft. Now, after incorporating the above formulas into the ultimate water-cut equation (as shown in Appendix A), a new model of ultimate water-cut is developed, given by,

$$WCult = (1 - {}^{q_{cr}}\!/_{Q}) \frac{\dfrac{Mh_w}{\ln {}^{r_e}\!/_{r_w} + s_w}}{\dfrac{Mh_w}{\ln {}^{r_e}\!/_{r_w} + s_w} + \dfrac{h_o}{(\ln {}^{r_e}\!/_{r_w} + s_o)}} \tag{5}$$

Validation of the proposed models using experiments

For simulation experiments, a 2-D radial-cylindrical model is built with IMEX simulation model depicted in Figure 3 using the base case reservoir properties, PVT and simulation grid data presented in Appendix C. In the model, transition zone is neglected and the produced oil and water is injected back to the oil drainage boundary and aquifer respectively at the constant pressure boundary (representing BOR boundary). The production well is completed in 50% of the total oil-zone thickness.

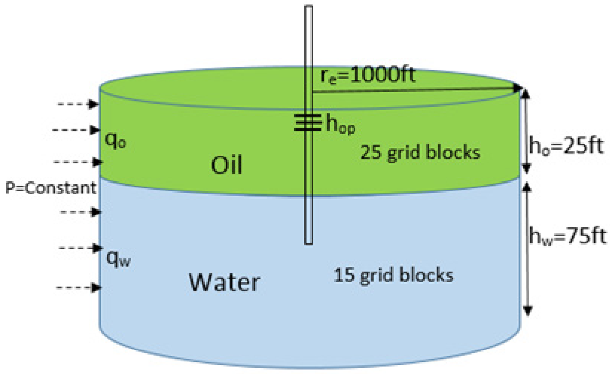

Figure 3 Radial model of oil with bottom water.

We compare the ultimate water-cut values from Eq. (2) and Eq. (5) with the the design of simulated experiments shown in Table 2 representing wide variety of reservoir/bottom-water systems. For creating matrix of experiments, we use the 3-level Box-Behnken design[8,9] to consider any non-linearity of the factors in the design. Three-levels (low, intermediate and high) of the reservoir parameters are chosed based on the practical field range values of reservoir properties: Mobility, horizontal permeability, aquifer thickness, penetration ratio and

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anisotropy ratio, as shown in Table 1. For 5 parameters chosen in this study, the design stipulates 46 number of runs (reservoir systems).

Critical-rate values, q_{cr} , for different reservoir systems used in Eq. (5) are estimated using Eq. A-12.

Table 1 Three-level values of different reservoir/aquifer system parameters

Levels	Mobility (M)	Aquifer thickness (h_w)	Horizontal permeability (k_h)	Penetration ratio ($\frac{h_{op}}{h_o}$)	Anisotropy ratio ($\frac{k_v}{k_h}$)
Low (-1)	1	20	50	0.2	0.01
Intermediate (0)	3	75	100	0.5	0.1
High (+1)	10	500	500	0.8	1

Table 2 Simulated and predicted data (WCult, oil-rate and water-rate) for an experimental matrix: $h_o = 25\,ft$; $Q = 2000 \, {}^{bbl}\!/_{day}$

Reservoir-system #	Mobility (M)	Aquifer thickness, (h_w)	Horizontal perm. (k_h)	Penetration ratio ($\frac{h_{op}}{h_o}$)	Anisotropy ratio, $\frac{k_v}{k_h}$	Simulated WCult	WCult (From Eq. 2)	WCult (From Eq. 5)	Abs. Discrepancy (Eq. 2 and 5)	Pressure drawdown ($p_e - p_w$)	Simulated oil-rate	Simulated water-rate	Predicted Oil-rate (From Eq. 3)	Predicted water-rate (From Eq. 4)
1	10	75	100	0.5	1.0	0.968	0.967	0.958	0.010	609	64	1936	70	1940
2	1	75	100	0.5	0.0	0.720	0.745	0.713	0.046	680	560	1440	480	1460
3	10	20	100	0.5	0.1	0.902	0.888	0.891	0.003	1178	196	1804	182	1800
4	10	75	500	0.5	0.1	0.959	0.966	0.958	0.007	152	82	1918	67	1950
5	1	75	50	0.5	0.1	0.720	0.748	0.708	0.057	1147	560	1440	501	1470
6	3	75	50	0.8	0.1	0.905	0.900	0.884	0.018	962	190	1810	196	1790
7	3	75	100	0.5	0.1	0.903	0.899	0.879	0.023	702	194	1806	205	1800
8	1	20	100	0.5	0.1	0.465	0.442	0.450	0.017	629	1070	930	970	960
9	10	500	100	0.5	0.1	0.974	0.995	0.989	0.006	625	52	1948	19	1990
10	3	500	100	0.5	1.0	0.965	0.982	0.946	0.038	480	70	1930	90	1940
11	3	75	100	0.5	0.1	0.903	0.899	0.879	0.023	710	194	1806	207	1820
12	3	75	100	0.8	1.0	0.909	0.899	0.880	0.022	410	182	1818	206	1820
13	3	75	50	0.5	1.0	0.916	0.899	0.873	0.030	1137	168	1832	218	1810
14	10	75	100	0.2	0.1	0.968	0.967	0.957	0.011	1535	64	1936	72	1940
15	3	20	500	0.5	0.1	0.726	0.701	0.707	0.009	194	548	1452	498	1480
16	10	75	100	0.8	0.1	0.968	0.968	0.962	0.006	524	64	1936	64	1950
17	3	500	100	0.5	0.0	0.920	0.982	0.968	0.014	716	160	1840	48	1880
18	10	75	50	0.5	0.1	0.963	0.968	0.960	0.007	1490	74	1926	65	1910
19	3	75	500	0.5	1.0	0.898	0.894	0.868	0.030	114	204	1796	218	1810
20	3	75	50	0.5	0.0	0.908	0.899	0.883	0.018	1696	184	1816	200	1820
21	3	20	100	0.8	0.1	0.753	0.705	0.710	0.006	731	494	1506	522	1535
22	3	75	100	0.2	0.0	0.887	0.898	0.876	0.025	1805	226	1774	209	1810
23	3	75	100	0.8	0.0	0.887	0.899	0.886	0.015	565	226	1774	193	1810
24	3	20	50	0.5	0.1	0.768	0.705	0.712	0.009	2043	464	1536	525	1560
25	1	500	100	0.5	0.1	0.904	0.948	0.895	0.059	575	192	1808	170	1830
26	3	75	500	0.5	0.0	0.865	0.891	0.874	0.018	166	270	1730	195	1780
27	3	75	500	0.8	0.1	0.891	0.897	0.881	0.018	97	218	1782	198	1810
28	1	75	100	0.8	0.1	0.720	0.748	0.716	0.045	395	560	1440	483	1470
29	3	75	100	0.5	0.1	0.903	0.899	0.879	0.023	714	194	1806	208	1830
30	3	20	100	0.5	1.0	0.755	0.705	0.712	0.011	846	490	1510	515	1540

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(Table 2 continue..)

31	10	75	100	0.5	0.0	0.944	0.967	0.961	0.006	899	112	1888	63	1930
32	3	20	100	0.2	0.1	0.753	0.705	0.715	0.014	1890	494	1506	507	1535
33	3	75	50	0.2	0.1	0.905	0.899	0.870	0.033	2921	190	1810	227	1845
34	3	500	100	0.2	0.1	0.946	0.982	0.949	0.034	1244	108	1892	82	1910
35	1	75	100	0.2	0.1	0.700	0.746	0.689	0.083	1132	600	1400	528	1430
36	3	75	100	0.5	0.1	0.903	0.899	0.879	0.023	714	194	1806	208	1830
37	3	75	100	0.5	0.1	0.903	0.899	0.879	0.023	718	194	1806	209	1840
38	3	500	50	0.5	0.1	0.947	0.983	0.963	0.020	1218	106	1894	60	1940
39	1	75	100	0.5	1.0	0.710	0.747	0.695	0.075	456	580	1420	523	1450
40	3	75	100	0.5	0.1	0.903	0.899	0.879	0.023	714	194	1806	208	1830
41	3	500	500	0.5	0.1	0.938	0.976	0.957	0.020	121	124	1876	60	1920
42	3	20	100	0.5	0.0	0.755	0.704	0.710	0.008	1163	490	1510	517	1530
43	3	500	100	0.8	0.1	0.946	0.983	0.967	0.016	413	108	1892	54	1940
44	1	75	500	0.5	0.1	0.700	0.733	0.693	0.057	112	600	1400	488	1430
45	3	75	100	0.2	1.0	0.909	0.898	0.855	0.051	1077	182	1818	256	1840
46	3	75	500	0.2	0.1	0.891	0.891	0.863	0.033	287	218	1782	224	1815

Using the pressure drawdown simulation data for different runs, oil and water production-rates were calculated using Eqs. (3) and (4) as shown in Table 2, which were then subsequently compared with their simulated data (from Table 2) shown in Figures 4 and 5. Near unit-slope correlation plot and high R2 value close to 1, approve the validity of underlying assumptions of these proposed models (Eqs. (3) and (4)) to a larger extent. The slight discrepancy is due to the assumptions of 1) piston-like displacement process and 2) displaced water completion as shown in Figure 2 that neglects the additional skin due to water inflow from aquifer to the oil-zone. Further, the comparison plot between the predicted values of WCult from Eqs. (2) and (5) and the simulated values (from Table 2) is shown in Figure 6.

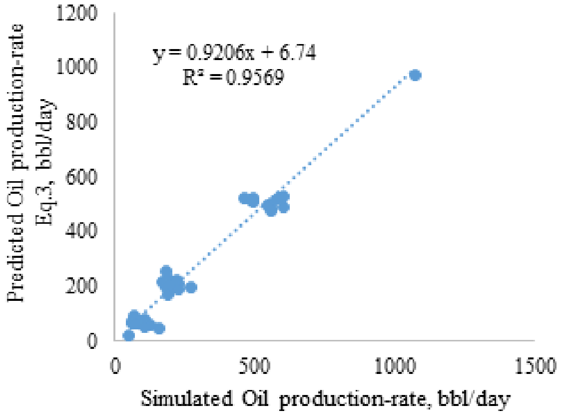

Figure 4 Simulated vs. predicted oil production rate (Eq. 3).

It is clear from the unit-slope correlation plot (Figure 6) that both the formulas give practically the same result. This infers that though the formula 2 ignores the inevitable non-radial flow to a partially penetration well, it still manages to conform to a more realistic physics-based formula 5 and hence predict the simulated WCult value.

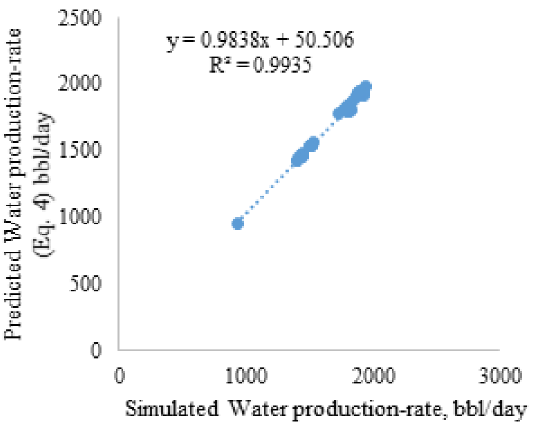

Figure 5 Simulated vs. predicted water production rate (Eq. 4).

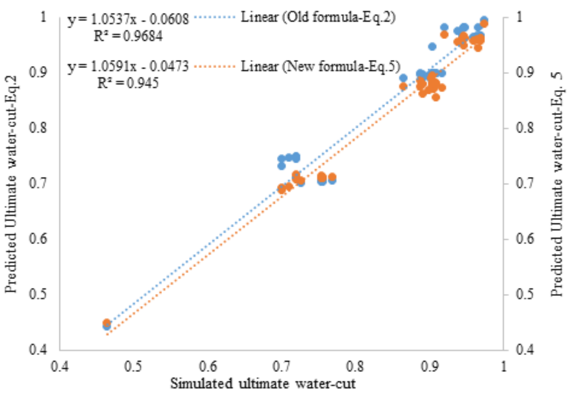

Figure 6 Simulated vs predicted ultimate water-cut with Eq. (2) and Eq. (5).

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Figure 7a shows the average absolute discrepancy (error), in percentage between the presently-used formula 2 and the proposed formula 5 using the data from Table 2. Also, Figure 7b shows the discrepancy between the formulas Eq. (2) and Eq. (5) for light oil reservoirs (M<3). From these two figures, it can be inferred that for the light oil reservoirs (when the mobility ratio is <3), the theoretical formula 2 may significantly deviate from the better (physically accurate) formula 5 for some cases (Figure 7a) with discrepancy as high as 8% (Figure 7b), which may not be reflected in Figure 6 due to considerable wide variety of sample size. In this study, any discrepancy exceeding the limit of 5% would be considered significant. This implies that for the light oil reservoir, the simplified assumptions of formula 2 may no longer allow it to better predict the actual WCult values, for which the formula 5 can serve better. This can be also be justified by the mathematical proof in Appendix B. So, in practice, formula 5 should be preferred for general use.

On the other hand, for moderate to high mobility ratio reservoirs ($M \geq 3$), Figure 7a shows that the average discrepancy between the formulas is less than 5%, which is insignificant. This implies that in those conditions, formula (5) can be reduced to formula (2), which is also shown mathematically in Appendix B. So, Eq. (2), being simpler than Eq. (5), suffices to predict WCult for viscous oil reservoirs ($M \geq 3$).

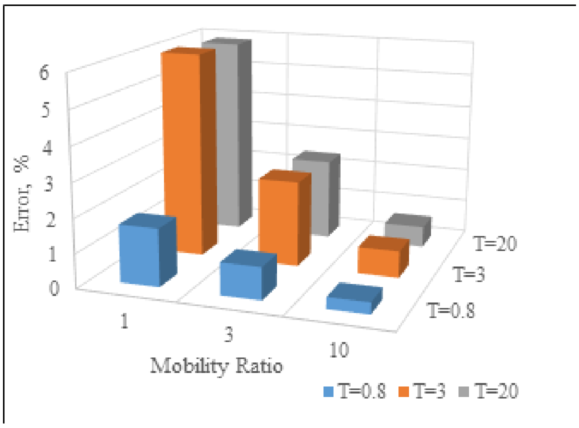

Figure 7a Average absolute discrepancy, in % between formulas 5 and 2.

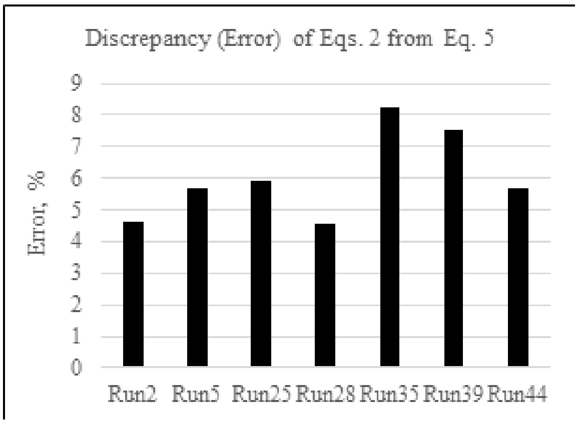

Figure 7b Absolute Discrepancy, in % between formulas 5 and 2 for runs having M<3.

Conclusions

Results of the study are summarized in the following conclusions:

1. A new analytical formula for WCult has been proposed including the physical effect ignored in the presently-used formula: partial penetration of oil zone, and aquifer. The formula utilizes the new models of oil and water production-rates during the ultimate water-cut stage. The derivation of models considers the piston-like displacement process and the inflow of oil and water into separate completions at the top of oil-zone and aquifer respectively.
2. The proposed formulas are systematically verified for wide variety of reservoir systems using design of simulated experiments (IMEX). High R2 value for the plot between the simulated and the predicted oil and water production-rates approves the validity of the proposed model's underlying assumptions to a large extent. However, sight discrepancy can be attributed to the above assumptions.
3. In general, both the formulas (proposed and presently-used) of WCult predicts almost the same results which matches the simulated WCult values. However, for the light oil reservoirs (mobility ratio<3), simulations showed that the theoretical presently used-formula may significantly deviate from the (physically accurate) proposed formula. This is also confirmed by mathematical proof, so in practice, proposed formula should be preferred for the possible avoidance of errors.
4. On the other hand, for viscous oil reservoirs (Mobility ratio≥3), comparison of the simulations with the predicted values showed that the presently-used formula suffices to predict the WCult values. This fact that the proposed formula reduces to presently-used formula for the above reservoirs, can be justified mathematically.

Nomenclature

μ_o = viscosity of oil, cp

μ_w = viscosity of water, cp

$\Delta\rho$ = density difference between water and oil, lb/ft^3

B_o = oil formation volume factor, bbl/stb

B_w = water formation volume factor, bbl/stb

BOR = balanced-oil-rate

h_o = oil-zone thickness, ft

h_{op} = perforated length, ft

h_{opo} = length of well-completion occupied by oil during WCult stage, ft

h_{opw} = length of well-completion occupied by water during WCult stage, ft

h_w = aquifer thickness, ft

k_h = horizontal permeability, md

k_o = effective permeability of oil, md

k_{ro} = relative permeability of oil

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k_{rw} = relative permeability of water

k_v/k_h =Anisotropy ratio, fraction

k_w = effective permeability of water, md

M = mobility ratio between water and oil, fraction

p_e = reservoir pressure, psi

p_w = well-bottomhole pressure, psi

q_{cr} =critical oil rate, bbl/day

q_o = oil flow rate, bbl/day

q_w = water flow rate, bbl/day

Q = Total production rate, bbl/day

r_w = wellbore radius, ft

r_e = reservoir radius, ft

S_o = Partial penetration skin due to oil-inflow

S_w = Partial penetration skin due to water-inflow

T = Ratio of aquifer thickness to oil-zone thickness

WC = water-cut, fraction

$WCult$ = Ultimate water cut, fraction

Appendix A: Derivation of new analytical WCult formula

Assuming piston-like displacement process, the rise of water cone before final stabilization covers larger area of oil completion. Eventually, the ratio of well completion producing oil and water becomes equal to the ratio of oil and water zone thickness, when ultimate water-cut is reached.[3] So, the length of well-completion occupied by oil during WCult stage:

$$h_{opo} = \frac{h_o}{h_o + h_w} \times h_{op} \tag{A-1}$$

And, the length of well-completion occupied by water during WCult stage:

$$h_{opw} = \frac{h_w}{h_o + h_w} \times h_{op} \tag{A-2}$$

This follows that the well completion system during water cone stabilization stage can be assumed to be the combination of the oil completion (producing only oil) at the top of oil-zone and the displaced water completion (producing only water) at the top of aquifer (Figure 2). So, oil inflow rate due to partial penetration in oil-zone (producing only oil) is given by,

$$q_o = \frac{2\pi k_o h_o (p_e - p_w)}{\mu_o (\ln {}^{r_e}\!/\!_{r_w} + so)}$$

Since, $k_o = k_h k_{ro}$, we get:

$$q_o = \frac{2\pi k_h k_{ro} h_o (p_e - p_w)}{\mu_o (\ln {}^{r_e}\!/\!_{r_w} + so)} \tag{A-3}$$

Where, s_o is the skin factor[10] due to oil-inflow and is given by,

$$S_o = (\frac{1}{h_{opD}} - 1)\ln\frac{\pi}{2r_{oD}} + \frac{1}{h_{opD}}\ln\left[\frac{h_{opD}}{2 + h_{opD}}\left(\frac{A-1}{B-1}\right)^{1/2}\right] \tag{A-4}$$

$$h_{opD} = {}^{h_{opo}}\!/\!_{h_o} = \frac{h_{op}}{h_o + h_w} \qquad \text{(From Eq. (A-1))} \tag{A-5}$$

$$r_{oD} = \left({}^{r_w}\!/\!_{h_o}\right)\left({}^{k_v}\!/\!_{k_h}\right)^{1/2}; \quad A = 4/h_{opD}\ ; \ B = 4/3h_{opD}$$

Now, again water inflow rate due to partial penetration in an aquifer (producing only water) is given by,

$$q_w = \frac{2\pi k_w h_w (p_e - p_w)}{\mu_w (\ln {}^{r_e}\!/\!_{r_w} + s_w)}$$

Since, $k_w = k_h k_{rw}$, we get:

$$q_w = \frac{2\pi k_h k_{rw} h_w (p_e - p_w)}{\mu_w (\ln {}^{r_e}\!/\!_{r_w} + s_w)} \tag{A-6}$$

So, the skin factor, S_w due to water-inflow can be represented by[10]:

$$S_w = (\frac{1}{h_{wpD}} - 1)\ln\frac{\pi}{2r_{wD}} + \frac{1}{h_{wpD}}\ln\left[\frac{h_{wpD}}{2 + h_{wpD}}\left(\frac{Aw-1}{Bw-1}\right)^{1/2}\right] \tag{A-7}$$

$$h_{wpD} = {}^{h_{opw}}\!/\!_{h_w} = \frac{h_{op}}{h_o + h_w} \qquad \text{(From Eq. (A-2))} \tag{A-8}$$

$$r_{wD} = \left({}^{r_w}\!/\!_{h_w}\right)\left({}^{k_v}\!/\!_{k_h}\right)^{1/2}; \quad Aw = 4/h_{wpD}\ ; \quad Bw = 4/3h_{wpD}$$

From Eqs. (A-5) and (A-8), we get:

$$h_{wpD} = h_{opD} = h_{pD} \tag{A-9}$$

Ultimate Water-cut, during water-cut stabilization stage[3] is given by:

$$WCult = \left(1 - \frac{q_{cr}}{Q}\right)\frac{q_w}{q_w + q_o} = \left(1 - \frac{q_{cr}}{Q}\right)\frac{1}{1 + \frac{q_o}{q_w}} \tag{A-10}$$

Substituting q_o and q_w from Eqs. (A-3) and (A-6) in (A-10), we get:

$$WCult = \left(1 - \frac{q_{cr}}{Q}\right)\frac{1}{1 + \dfrac{\dfrac{2\pi k_h k_{ro} h_o (p_e - p_w)}{\mu_o (\ln {}^{r_e}\!/\!_{r_w} + so)}}{\dfrac{2\pi k_w h_w (p_e - p_w)}{\mu_w \left[\ln {}^{r_e}\!/\!_{r_w} + s_w\right]}}} = \left(1 - \frac{q_{cr}}{Q}\right)\frac{\dfrac{Mh_w}{\ln {}^{r_e}\!/\!_{r_w} + s_w}}{\dfrac{Mh_w}{\ln {}^{r_e}\!/\!_{r_w} + s_w} + \dfrac{h_o}{\left(\ln {}^{r_e}\!/\!_{r_w} + s_o\right)}} \tag{A-11}$$

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Where, $M = \dfrac{k_{rw}/\mu_w}{k_{ro}/\mu_o}$ Critical rate, q_{cr} in above Eq. (A-11) can be substituted by the following formula[11]:

$$q_{cr} = 0.0783\times10^{-4}\left[\frac{\Delta\rho k_o\left(h_o^{\,2}-h_{op}^{\,2}\right)}{\mu_o B_o}\right]\left[0.7311+\frac{1.943}{\dfrac{r_e}{h_o}\sqrt{\dfrac{k_v}{k_h}}}\right] \qquad \text{(A-12)}$$

Where, all the parameters are in field units.

Appendix B: Mathematical convergence of new formula to presently-used formula

Using Eqs. (A-4), (A-7) and (A-9), Eq. 5 can be rewritten as:

$$\left(1-\frac{q_{cr}}{Q}\right)\frac{\dfrac{Mh_w}{\ln{}^{r_e}\!/_{r_w}+s_w}}{\dfrac{Mh_w}{\ln{}^{r_e}\!/_{r_w}+s_w}+\dfrac{h_o}{\left(\ln{}^{r_e}\!/_{r_w}+s_o\right)}} = \left(1-\frac{q_{cr}}{Q}\right)\frac{\dfrac{Mh_w}{\ln{}^{r_e}\!/_{r_w}+\left(\frac{1}{h_{pD}}-1\right)\ln\dfrac{\pi h_w}{2r_w\left({}^{k_v}\!/_{k_h}\right)^{1/2}}+\frac{1}{h_{pD}}\ln\left[\frac{h_{opD}}{2+h_{pD}}\left(\frac{A-1}{B-1}\right)^{1/2}\right]}}{\dfrac{Mh_w}{\left(l\,n{}^{r_e}\!/_{r_w}+\left(\frac{1}{h_{pD}}-1\right)\ln\dfrac{\pi h_w}{2r_w\left({}^{k_v}\!/_{k_h}\right)^{1/2}}+\frac{1}{h_{pD}}\ln\left[\frac{h_{opD}}{2+h_{pD}}\left(\frac{A-1}{B-1}\right)^{1/2}\right]\right)}+\dfrac{h_o}{\left(l\,n{}^{r_e}\!/_{r_w}+\left(\frac{1}{h_{pD}}-1\right)\ln\dfrac{\pi h_o}{2r_w\left({}^{k_v}\!/_{k_h}\right)^{1/2}}+\frac{1}{h_{pD}}\ln\left[\frac{h_{opD}}{2+h_{pD}}\left(\frac{A-1}{B-1}\right)^{1/2}\right]\right)}} = \left(1-\frac{q_{cr}}{Q}\right)\frac{Mh_w}{Mh_w+h_o\times\dfrac{\left(l\,n{}^{r_e}\!/_{r_w}+\left(\frac{1}{h_{pD}}-1\right)\ln\dfrac{\pi h_w}{2r_w\left({}^{k_v}\!/_{k_h}\right)^{1/2}}+\frac{1}{h_{pD}}\ln\left[\frac{h_{opD}}{2+h_{pD}}\left(\frac{A-1}{B-1}\right)^{1/2}\right]\right)}{\left(l\,n{}^{r_e}\!/_{r_w}+\left(\frac{1}{h_{pD}}-1\right)\ln\dfrac{\pi h_o}{2r_w\left({}^{k_v}\!/_{k_h}\right)^{1/2}}+\frac{1}{h_{pD}}\ln\left[\frac{h_{opD}}{2+h_{pD}}\left(\frac{A-1}{B-1}\right)^{1/2}\right]\right)}} \qquad \text{(B-1)}$$

Substituting $T = {}^{h_w}\!/_{h_o}$, and $C = \ln{}^{r_e}\!/_{r_w}+\left(\dfrac{1}{h_{pD}}-1\right)\ln\dfrac{\pi h_o}{2r_w\left({}^{k_v}\!/_{k_h}\right)^{1/2}}+\dfrac{1}{h_{pD}}\ln\left[\dfrac{h_{opD}}{2+h_{pD}}\left(\dfrac{A-1}{B-1}\right)^{1/2}\right]$ in Eq. (B-1), we get:

$$\left(1-\frac{q_{cr}}{Q}\right)\frac{\dfrac{Mh_w}{\ln{}^{r_e}\!/_{r_w}+s_w}}{\dfrac{Mh_w}{\ln{}^{r_e}\!/_{r_w}+s_w}+\dfrac{h_o}{\left(\ln{}^{r_e}\!/_{r_w}+s_o\right)}} = \left(1-\frac{q_{cr}}{Q}\right)\frac{MTh_o}{MTh_o+h_o\times\dfrac{\left(C+\left(\dfrac{1}{h_{pD}}-1\right)\ln T\right)}{C}} = \left(1-\frac{q_{cr}}{Q}\right)\frac{Mh_o}{h_o\times\left({}^{1}\!/_{T}+\left(M+\dfrac{\frac{\ln T}{T}}{C\Big/\left(\dfrac{1}{h_{pD}}-1\right)}\right)\right)} \qquad \text{(B-2)}$$

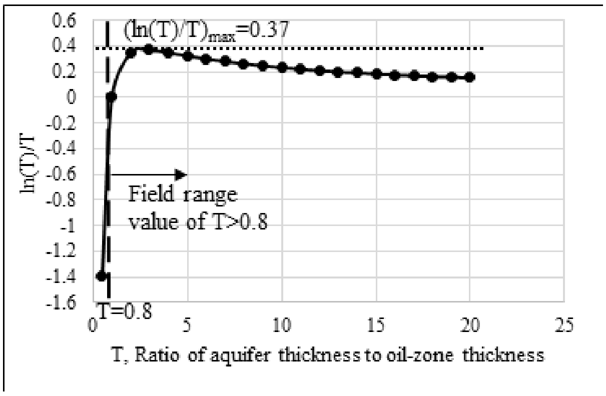

Figure B-1 Pattern graph of log(T)/T vs. T; (T=ratio of aquifer thickness to oil-zone thickness).

Figure B-1 clearly shows the maximum value of $\dfrac{\ln T}{T}$ is 0.37. Subsequently, the approximate maximum possible value of $\dfrac{\frac{\ln T}{T}}{C\Big/\left(\dfrac{1}{h_{pD}}-1\right)}$ is 0.15 for the practical field operating range values of h_{pD} (between 0.1 and 1) and for practical value of T (>0.8). Minimum possible value of $\dfrac{1}{r}$ tends to 0 for infinite thick aquifers.

Now, assuming 5% maximum possible error is permissible in predicted WCult value given by Eq. (B-2); for viscous reservoirs (when mobility ratio ≥ 3), any value of $\dfrac{\frac{\ln T}{T}}{C\Big/\left(\dfrac{1}{h_{pD}}-1\right)}$ would lie withing this error margin of Eq. (B-2) and hence, the part ‘$\dfrac{\frac{\ln T}{T}}{C\Big/\left(\dfrac{1}{h_{pD}}-1\right)}$’ can be ignored. So, Eq. (B-2) or Eq. (5) can be rewritten as:

$$\left(1-\frac{q_{cr}}{Q}\right)\frac{\dfrac{Mh_w}{\ln{}^{r_e}\!/_{r_w}+s_w}}{\dfrac{Mh_w}{\ln{}^{r_e}\!/_{r_w}+s_w}+\dfrac{h_o}{\left(\ln{}^{r_e}\!/_{r_w}+s_o\right)}} = \left(1-\frac{q_{cr}}{Q}\right)\frac{Mh_o}{Mh_o+{}^{h_o}\!/_{T}} \qquad \text{(B-3)}$$

$$= \left(1-\frac{q_{cr}}{Q}\right)\frac{Mh_w}{Mh_w+h_o}$$

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Above derivation mathematically proves that Eq. (5) reduces to Eq. (2) in case of viscous oil reservoirs. However, for mobility ratio<3, Eq. (5) may or may not reduce to Eq. (2) depending upon the ratio of aquifer to oil-zone thickness.

Appendix C: Complete Reservoir Simulation Input Data

Table C-1 Reservoir and Well Input data

Parameter	Unit	Value
Datum depth	ft	5000
Thickness of oil zone	ft	25
Depth of WOC	ft	5025
Thickness of water zone	ft	75, varied
Reservoir pressure at datum depth	psi	6000
Position of top completion from formation top	ft	0
Perforated length	ft	12, varied
Horizontal permeability	md	100, varied
Anisotropy ratio	md	0.1, varied
Porosity	fraction	0.3
Well radius	ft	0.25
Outer radius of oil-zone	ft	1000
Outer radius of water zone	ft	1000
Total liquid Production rate	bpd	2000

Table C-2 Fluid Properties Input Data

Property	Unit	Value
Reference pressure	psi	6000
Formation oil volume factor	rb/stb	1.2
Relative oil permeability at connate water saturation	fraction	1
Water compressibility	1/psi	3.3202e--6
Oil compressibility	1/psi	1.50E-05
water viscosity	cp	0.5
Oil viscosity	cp	1.5, varied
oil density	lb/cuft	43.65
Water density	lb/cuft	60.55
Bubble point	psi	100

Table C-3 Simulation Grid Data

Region	Direction	Grid Number
Oil zone	R	20
	Φ	1
	Z	25
Water zone	R	29
	Φ	1
	Z	15

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