**New exercises – competition B** (see page 161): **B. 5302.** Either +1 or −1 is written in every field of an $8\times 8$ table so that the sum of all entries is 0. The sum of the numbers is calculated for each row and column. What is the maximum possible number of positive sums out of the 16 sums obtained in this way? (*3 points*) (Based on the idea of *M. E. Gáspár,* Budapest) **B. 5303.** The isosceles right-angled triangle $ABC$ has its right angle at $C$. $D$ is an interior point of side $BC$ such that the angle $CDA$ is $75^\circ$. Given that triangle $ADC$ has unit area, prove that $BD=2$. (*4 points*) (Proposed by *M. Hujter,* Budapest) **B. 5304.** *a*) Are there positive integers $a$ and $b$ such that $a+b\mid a^2+b^2$, but $a+b\nmid a^4+b^4$? *b*) Are there positive integers $a$ and $b$ such that $a+b\mid a^4+b^4$, but $a+b\nmid a^2+b^2$? (*4 points*) (Proposed by *B. Hujter,* Budapest) **B. 5305.** Let $A_1$ and $A_2$, respectively, denote the points lying closer to $B$ and closer to $C$ that divide side $BC$ of a triangle $ABC$ in a 1:2 ratio. Define points $B_1$ and $B_2$ on side $CA$, and points $C_1$ and $C_2$ on side $AB$ in an analogous way. Prove that the centroid of triangle $ABC$ lies on the line joining the common points of the circumscribed circles of triangles $A_1B_1C_1$ and $A_2B_2C_2$. (*4 points*) (Proposed by *B. Bíró,* Eger) **B. 5306.** We have a weighted (six-sided) die and a weighted coin. On one side of the coin there is one dot, and on the other side there are two dots. The expected value of the number of dots appearing on top is the same for the die and the coin. Show that if the die and the coin are thrown simultaneously then the probability of getting more dots on the coin than on the die is greater than the probability of getting more dots on the die than on the coin. (*5 points*) (Proposed by *V. Vígh,* Sándorfalva) **B. 5307.** The area of an acute-angled triangle is $T$, its inradius is $r$, and its circumradius is $R$. Show that $\sqrt{3}T\leqslant (r+R)^2$. (*5 points*) (Proposed by *L. B. Simon,* Budapest) **B. 5308.** Let $a_n$ denote the least common multiple of the positive integers $n+1, n+2, \ldots, n+10$. Find the greatest real number $\lambda$ for which $\lambda a_n\leqslant a_{n+1}$ is always true. (*6 points*) (Proposed by *P. P. Pach,* Budapest) **B. 5309.** Given the axis and two points of a parabola, construct its focus and directrix. (*6 points*) (Proposed by *G. Holló,* Budapest)

**New problems – competition A** (see page 162): **A. 848.** Let $G$ be a planar graph, which is also bipartite. Is it always possible to assign a vertex to each face of the graph such that no two faces have the same vertex assigned to them? (Submitted by *Dávid Matolcsi,* Budapest) **A. 849.** For real number $r$ let $f(r)$ denote the integer that is the closest to $r$ (if the fractional part of $r$ is 1/2, let $f(r)$ be $r-1/2$). Let $a>b>c$ rational numbers such that for all integers $n$ the following is true: $f(na)+f(nb)+f(nc)=n$. What can be the values of $a$, $b$ and $c$? (Submitted by *Gábor Damásdi,* Budapest) **A. 850.** Prove that there exists a positive real number $N$ such that for arbitrary real numbers $a,b>N$ it is possible to cover the perimeter of a rectangle with side lengths $a$ and $b$ using non-overlapping unit disks (the unit disks can be tangent to each other). (Submitted by *Benedek Váli,* Budapest)

## Problems in Physics

(see page 187)

**M. 421.** Draw lines on a piece of paper with a soft black graphite pencil. We can assume that the graphite is “smeared” in atomic layers, and that the distance between adjacent atomic layers is 0.34 nm. Determine the height of a line in terms of number of carbon atoms above each other.

**G. 809.** We rub a small object with a light file in a horizontal plane. The line of action of the sum of the forces exerted by our two hands passes through the centre of the filed surface of the object, and this line of action makes an angle of $30^\circ$ with the vertical. What is the coefficient of friction between the file and the object? **G. 810.** When a penalty kick is shot, the average speed of the ball can reach a speed of 150 km/h. How much time does the goalkeeper have to save the penalty if he is in the middle of the goal at the instant when the ball is kicked and the ball is moving towards one of the bottom corners of the goal? Is the following statement true: *"You can't defend a penalty kick well, they can only kick it badly."*? **G. 811.** Three rectangular blocks are pulled along a horizontal sheet, with a constant force $F$, as shown in the *figure.* The blocks are moving along a straight line all the time, and their instantaneous velocity is $v_0$. How does the tension in the threads connecting the blocks depend on the value of the coefficient of kinetic friction $\mu$ between the blocks and the sheet? **G. 812.** A body immersed into water can be kept in equilibrium with a force of 1.5 N, and with a force 1 N when it is immersed into glycerine. What is the volume and density of the body?

**P. 5472.** How long does it take the James Webb space telescope to orbit the Sun once? **P. 5473.** A projectile of mass $M=3$ kg is fired vertically at a speed of $v=50$ m/s, and it explodes into two parts after $t=3$ s. The piece with mass $m_1=1$ kg will land in $t_1=1$ s. *a*) How long after the explosion will the other piece hit the ground? *b*) If the first piece has landed 40 m from the firing position, how far apart will the two pieces be after the other piece has also landed? **P. 5474.** One end of a uniform density, thin stick of mass $m$ and of length $\ell$ is fixed with a hinge on a horizontal surface. The other end is struck by a brief horizontal force of magnitude $F$, which is perpendicular to the rod. At this instant what is the acceleration of the centre of the stick, what is its angular acceleration and what is the force exerted by the hinge on the stick? **P. 5475.** A container with mass $M=32$ kg and volume $V=4\ \mathrm{dm}^3$ can move frictionlessly on a horizontal table. It is divided into two parts by a piston of mass $m=16$ kg. On the left side of the piston there is a mixture of gases of volume $V_0=1\ \mathrm{dm}^3$, at a pressure of $p_0=0.3$ MPa, and of adiabatic exponent $\kappa=1.5$. On the right side of the piston there is vacuum. What is the relative velocity at which the piston will hit the wall of the cylinder if the piston is released? Assume that the gas is in thermal equilibrium for all the time. **P. 5476.** Three alike metal plates of area $A$ are placed parallel to each other. The distance between the plates is small compared to the size of the plates. *a*) What is the electric field strength between the plates if the charge of the plate on the left is $+Q$, the charge of the middle plate is $+2Q$ and that of the right plate is $+3Q$? *b*) What is the electric field strength between the plates if the charge of the plate on the left is $+Q$, the charge of the middle plate is $-2Q$ and that of the right plate is $+3Q$? **P. 5477.** An ideal battery of emf $U=24$ V, resistors of resistance values $R_1=500\ \Omega$ and $R_2=300\ \Omega$, a switch and an ideal transformer were used to construct the circuit shown in the *figure.* The primary coil of the transformer has a number of turns of $N_1=800$ and the secondary coil has $N_2=1000$ turns. The switch, which had been open for a long time, was once closed. What were the values of the current in the primary and secondary coils immediately after the switch was closed? **P. 5478.** A plano convex glass lens is bounded by water on its flat side and by air on its convex side. *a*) What is the ratio of the two focal lengths corresponding to the two sides of the lens? *b*) What will this ratio be if the two media at the sides of the lens are reversed? The lens is thin and has a small aperture angle. The refractive index of glass is 3/2 and that of water is 4/3. **P. 5479.** According to the classical electron model, the electron is a uniformly charged insulating spherical shell whose electrostatic energy is equal to the rest energy of the electron $mc^2$. Using the laws of classical mechanics, determine the kinetic energy that should be given to the electron if it is to collide with another initially stationary electron such that they "touch" each other? **P. 5480.** Through a given point $P$ of a vertical plane, slopes with different angles of inclination are laid (perpendicularly to the plane), and point-like objects, which were released from rest, slide down the planes. What is the locus of points which are reached by the sliding objects in a given time of $t$? The coefficient of friction between the slopes and the objects is $\mu$.