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SPECTRAL STRUCTURE OF REFLECTION SEISMOGRAMS FROM INSTANTANEOUS FREQUENCY DISPLAYS

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The spectral characteristic of seismograms often resembles processes with mixed spectra. Such processes are usually nonstationary. Instantaneous phase/instantaneous frequencies derived from analytical signal technique enable one to detect the quasiharmonics present in a seismogram. The method is valid irrespective of whether the process is stationary or not. Time evolution and decay of such quasiharmonics can also be estimated with sufficient accuracy from instantaneous frequency displays.

d: theoretical seismograms, energy spectrum, instantaneous frequency, Hilbert transform

1. Introduction

A signal or a seismic trace (time series) can be described by certain parameters such as envelope, instantaneous phase and "instantaneous frequency" (IF). These parameters have been applied in geological interpretations [Taner et al. 1979]. The envelope of a signal is related to its strength and the phase may be understood by the number of accumulated cycles starting from a given time. The IF is defined as the derivative of instantaneous phase (or phase velocity which can also be negative). Although it can be defined clearly in mathematical terms, it is obviously not the same as that of Fourier frequency. In fact an IF that is present in a signal can fail to show up in the Fourier decomposition of the signal. In certain cases the IF will have values for which the Fourier spectrum of the signal vanishes [Mandel 1974]. It has been pointed out that the frequency decomposition through the Fourier transform technique cannot describe the local non-stationarities in time whereas the IF plots emphasize these non-stationarities. We may state that IF plots represent a stochastic process where local non-stationarities in phase velocities are displayed. So, in general, one should be extremely cautious in associationg the IF with the usual frequency interpretation of seismic sections.

The derivatives of instantaneous phase (instantaneous frequency) with time of a reflection seismogram represent essentially a stochastic process. The term "instantaneous frequency" has been objected to by many authors for the reason that one cannot embrace simultaneously two mutually exclusive variables such as Fourier frequency and time. However, the so-called instantaneous frequency

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can be identified with Fourier frequency in certain cases and for narrow-band signals the mean square bandwidth is equal to the variance of the instantaneous frequency with appropriate weighting factor. This can be proved from signal energy distribution function with time and frequency. The distribution function directly leads to the well-known Fourier uncertainty and a short time spectrum can also be defined as proposed first by De Bruijn.

Most of the above aspects of IF are well-known. However one can show that statistical averages of Fourier frequency and IF, when appropriately weighted, are approximately equal (if the process has an envelope which is slowly varying with time). For example, the variance of IF with respect to Fourier mean frequency is related to the mean square bandwidth of the signal. We recall that when a wavemeter is tuned to a particular Fourier frequency with a modulated signal as input, we obtain a spike only when the IF runs through the Fourier frequency being measured [VAKMAN 1964]. From the principle of stationary phase it can be shown that the spectral density at any Fourier frequency is determined from the contributions of the integral during times when the IF coincides with the Fourier frequency. In this paper we will show that most of the relations connecting the IF and the Fourier frequency can be derived from the concept of signal energy density in time and frequency. Further, the spectral model of any complex process such as of a reflection seismogram can often be ascertained from its IF displays.

2. Signal representation

We represent a real signal or a seismic trace $f(t)$ by

$$f(t) = A(t)\cos\Theta(t) = A(t)\cos[\omega_0 t + \Phi(t)], \tag{1}$$

$A(t)$ and $\Theta(t)$ are the envelope and the phase respectively; may be termed as the slow part of the phase; ω_0 in the case of a seismic signal or seismic trace may be considered as the mean frequency about which the signal is centred in the frequency domain. In equation (1) the splitting of the signal into an amplitude part and an oscillatory part is obviously arbitrary and thus the representation is not unique. We consider next the complex representation of a signal:

$$\Psi(t) = f(t) + i\hat{f}(t) \tag{2}$$

$$= A(t)e^{i\Theta(t)} \tag{2a}$$

where

$$A(t) = \sqrt{f^2(t) + \hat{f}^2(t)} \tag{2b}$$

$$\Theta(t) = \arctan[\hat{f}(t)/f(t)] \tag{2c}$$

and "instantaneous frequency" is,

$$\omega(t) = \frac{d\Theta(t)}{dt} = \frac{f(t)\hat{f}'(t) - \hat{f}(t)f'(t)}{f^2(t)+\hat{f}^2(t)} \tag{2d}$$

To determine $\omega(t)$ we need not know $\Theta(t)$, if $\hat{f}(t)$ is known. We assume that $\hat{f}(t)$ is derivable through a linear operation on $f(t)$, viz. $\hat{f}(t) = Lf(t) = f(t)*k(t)$, where $k(t)$ is real operator and $*$ denotes convolution. It can be shown [MANDEL 1967] that $k(t)$ is uniquely determined when the signal is narrow band and is given by

$$k(t) = -\frac{1}{\pi} P\left(\frac{1}{t}\right) \tag{3}$$

or

$$\hat{f}(t) = \frac{1}{\pi} P \int \frac{f(\tau)}{\tau - t} d\tau \tag{4}$$

P denotes Cauchy's principal value. Henceforth limits of all integrals are from $-\infty$ to $+\infty$. The complex signal in equation (2) where $\hat{f}(t)$ is given by equation (4) is know as an analytic signal.

3. The "two frequencies"

Let us consider a vibration of the following form [VAKMAN 1973]:

$$f(t) = A_0 \sin^3 \omega_0 t \tag{5a}$$

The signal $\sin^3 \omega_0 t$ consist of two vibrations: $\sin \omega_0 t$ and $\sin 3\,\omega_0 t$. When we apply the analytic signal technique we obtain:

$$A(t) = A_0 \sqrt{\frac{5}{8} - \frac{3}{8}\cos 2\,\omega_0 t} \tag{5b}$$

$$\omega(t) = \frac{\frac{3}{4} - \frac{3}{4}\cos 2\,\omega_0 t}{\frac{5}{8} - \frac{3}{8}\cos 2\,\omega_0 t} \cdot \omega_0 \tag{5c}$$

We find that $A(t)$ changes from $1/2A_0$ to A_0 and $\omega(t)$ from 0 to $3/2\omega_0$ with a period of $1/2\omega_0$. We consider now a complex signal [MANDEL 1974]:

$$\Psi(t) = a_1 \exp\left[-i\left(\omega_0 - \frac{1}{2}\Delta\omega\right)t\right] + a_2 \exp\left[-i\left(\omega_0 + \frac{1}{2}\Delta\omega\right)t\right],$$
$$a_1 \neq a_2, \quad \Delta\omega \ll \omega_0 \tag{6a}$$

$\Psi(t)$ represents a combination of two sinusoidal oscillations with frequencies symmetrically placed with respect to ω_0. On application of the same technique:

$$\omega(t) = \omega_0 + \frac{1}{2}\Delta\omega\left(\frac{-a_1^2 + a_2^2}{a_1^2 + a_2^2 + 2a_1a_2 \cos \Delta\omega t}\right) \tag{6b}$$

When $\cos \Delta\omega t = 1$,

$$\omega(t) = \omega_0 + \frac{1}{2}\Delta\omega \frac{a_2 - a_1}{a_2 + a_1} \tag{7a}$$

and for $\cos \Delta\omega t = -1$,

$$\omega(t) = \omega_0 + \frac{1}{2}\Delta\omega \frac{a_2 + a_1}{a_2 - a_1}. \tag{7b}$$

It is obvious that the deflections of $\omega(t)$ about ω_0 are not symmetrical and no Fourier components are present with frequencies as high as given by equation (7b). However the distributions of "instantaneous frequencies" of a signal and the spectral distributions of the signals are related by:

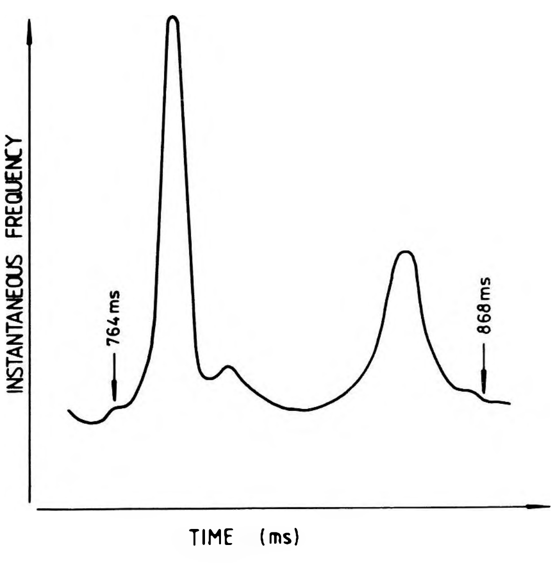

Fig. 1a. Instantaneous frequency of a segment of seismic trace: time window 764 ms—868 ms

1a. ábra. Egy szeizmogram szakasz pillanatnyi frekvencia függvénye. Időablak: 764 ms—868 ms

Рис. 1a. Мгновенная частота от сегмента сейсмической трассы: временное окно 764 мс—868 мс

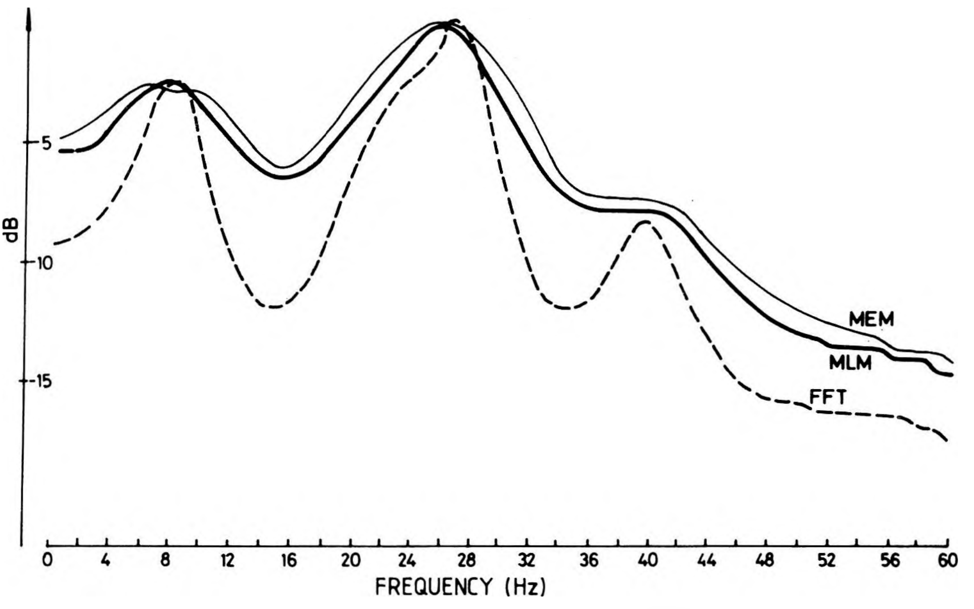

Fig. 1b. Spectral power estimate of the same segment of seismic trace
FFT–Fast Fourier Transform; MLM–Maxiumum Likelihood Method; MEM–Maximum Entropy Method

1b ábra. Ugyanazon szeizmogram szakasz spektrális energia eloszlása

Рис. 1b. Оценка спектральной мощности по сегменту сейсмической трассы: временное окно 764 мс—868 мс

$$\int (\omega-\omega_0)^2 \Psi(\omega)\mathrm{d}\omega = \int [\omega(t)-\omega_0]^2 A^2(t)\mathrm{d}t \tag{8a}$$

$\Psi(\omega)$ is the spectral density,

$$\int \Psi^2(\omega)\mathrm{d}\omega = \int A^2(t)\mathrm{d}t, \quad \text{by Parseval's theorem} \tag{8b}$$

The left hand side of equation (8a) is the mean square bandwidth. For convenience we assume the integrals in (8b) are equal to unity. The relation in equation (8a) is approximately true. One has to assume here a slowly varying envelope. Mandel [1974] has shown that this relation is also true for a stationary stochastic process. So there is no one to one relationship between these two "frequencies". This is because the two quantities are fundamentally different. *Figure 1a* shows sample by sample IF plots of a segment of one reflection seismic trace. The power spectral estimates for the same window are shown in *Figure 1b* where estimates have been calculated by different methods as per the prescription of LACOSS [1971]. It is seen that the peak frequency where the spectral power becomes maximum is absent in all the IF values within the same time window.

4. Sturm—Liouville problem

We recall that solutions of certain non-linear differential equations of Mathieu—Hill type represent oscillations with modulation in time of both amplitude and frequency. However these equations are not suitable for describing a seismic trace. Solutions of such equations often show that the maximum amplitude and minimum "instantaneous frequency" (maximum apparent period of one oscillation) occur together in time. This is not generally true for a seismic trace. We now deal with solutions of the following type of linear second-order homogeneous differential equation of Sturm—Liouville (SL) type:

$$U'' + a(t)U = 0 \tag{9}$$

The solutions of equation (9) will have three different regimes and being interested in the oscillatory regime we assume $a(t) = g^2(t)$. If $g(t)$ is sufficiently large we can obtain an approximate solution by the WBK method:

$$U(t) = \frac{A_0}{\sqrt{g(t)}} \exp\left[i \int g \mathrm{d}t - \Theta_0 \right] \tag{10}$$

The solution shows that the amplitude is closely coupled with "instantaneous frequency". In fact it is true for a large class of signals and to quote Cornelius LANCZOS [1961] "the amplitude of the vibration is always inversely proportional to the square root of the IF. The law of zeros in the oscillations of Bessel functions, Laguerre or Hermite type of polynomials is not independent of the law according to which maxima of successive oscillations change". We should note that the solution is not unique and the separation of amplitude and frequency can occur in infinitely many ways. We next consider the following SL equation:

$$\frac{\mathrm{d}^2 U(t)}{\mathrm{d}t^2} + [\omega(t) - \omega_0]^2 U(t) = 0 \tag{11}$$

where $U(t) = \Psi(t) \exp(-i\omega_0 t)$ is the complex envelope. The assumption of the narrow band nature of signals means that the variation of $\left|\frac{\mathrm{d}U}{\mathrm{d}t}\right|$ should be as small as possible. We recall the connection between SL equations and calculus of variations and consider the following quadratic functional [BELLMAN 1970]:

$$J(U) = \int (U'^2 - g^2(t)U^2)\mathrm{d}t \tag{12}$$

where $g^2(t) = [\omega(t) - \omega_0]^2$.

Any solution of the SL equation is a stationary point for the functional $J(U)$. We search for that particular U which furnishes the absolute minimum of $J(U)$ and in turn will also furnish us with a unique solution of (11). We write (12) as:

$$J(U) = \int \left\{ \left| \frac{\mathrm{d}U}{\mathrm{d}t} \right|^2 - [\omega(t) - \omega_0]^2 A^2(t) \right\} \mathrm{d}t \tag{13}$$

Since the fluctuation of the envelope should be minimum, our extremization problem of $J(U)$ reduces to the problem of finding the conditions for which

$$\int [\omega(t) - \omega_0]^2 A^2(t)\mathrm{d}t \quad \text{is minimum.}$$

From equation (8a) we have:

$$\int [\omega(t) - \omega_0]^2 A^2(t)\mathrm{d}t \cong \int (\omega - \omega_0)^2 \Psi(\omega)\mathrm{d}\omega,$$

$A(t)$ varies slowly with time. The right hand side of the above equation is the variance of spectral density $\Psi(\omega)$ and it will be minimum when he deviation of ω from ω_0 is minimum. This is satisfied when ω_0 is the mean Fourier frequency:

$$\omega_0 \frac{\int \omega(t)A^2(t)\mathrm{d}t}{\int A^2(t)\mathrm{d}t} = \frac{\int \omega \Psi(\omega)\mathrm{d}\omega}{\int \Psi(\omega)\mathrm{d}\omega} \tag{14}$$

So the first moment of $(\omega - \omega_0)$ vanishes. To obtain a unique complex envelope function satisfying SL equation (11), the variational problem reduces to:

$$\int (\omega - \omega_0)^2 \Psi(\omega)\mathrm{d}\omega = \text{minimum} \tag{15a}$$

with the auxiliary condition

$$\int (\omega - \omega_0)\Psi(\omega)\mathrm{d}\omega = 0 \tag{15b}$$

Now $\Psi(\omega)$ is given by MANDEL [1967]:

$$\Psi(\omega) = \frac{1}{4}\Phi(\omega)[1 + \mathrm{i}K(\omega)]^2 \tag{15c}$$

$\Phi(\omega)$ is the spectral density of the real signal $f_a(t)$ and $K(\omega)$ is the Fourier transform of $k(t)$. By applying standard variational technique, MANDEL [1967] has shown that the integral (15a) is indeed minimum if

$$K(\omega) = -\mathrm{i}\,\mathrm{sgn}\,\omega \tag{16}$$

or in time domain,

$$K(t) = -\frac{1}{\pi} P\left(\frac{1}{t}\right)$$

So we conclude that for a certain class of signals the complex envelope satisfying the SL equation possesses a unique solution when $\hat{f}(t)$ is the Hilbert transform of $f(t)$. Thus,

$$A(t) \approx \frac{A_0}{([\omega(t) - \omega_0]^2)\frac{1}{4}}. \tag{17}$$

The validity of this relation is confined to within the reqime where the WBK soution is acceptable. The motivation of introducing the Sturm—Liouville problem in describing oscillatory processes with varying amplitude and "varying frequency" is now evident. Equation (17) shoes that when the deviation of IF from the mean Fourier frequency is maximum we should expect a minimum of the envelope at that instant. *Figure 2* shows a plot of the evenlope with IF derived from a reflection trace. It shows that the main maxima and minima of the envelope may be predicted with accuracy from a study of IF displays only. The so-called IF is not only a fundamentally different quantity from Fourier frequency, it is also closely coupled with the envelope of the process. In other words the instantenous phase/frequency is not a robust attribute of the signal strength. This follows also from analytic signal theory. When we represent a complex signal such as $\Psi(t) = A(t) \exp(\mathrm{i}\Theta(t)$, $\Psi(t)$ being regular and analytic in the upper half of the complex t plane, the phase and logarithm of the envelope are in quadrature. We also note that two random variables at any particular instant may be orthogonal. But it does not mean that the two processes are uncorrelated [PAPOULIS 1965]. We have a similar situation in the present case.

5. Signal energy distribution in frequency and time

The Fourier method is used to describe a signal either in frequency domain or in time domain, the two domains being mutually exclusive. Evidently a term like "freqency varying with time" is contradictory in the Fourier sense. To describe a signal energy density in frequency and time we attempt to combine

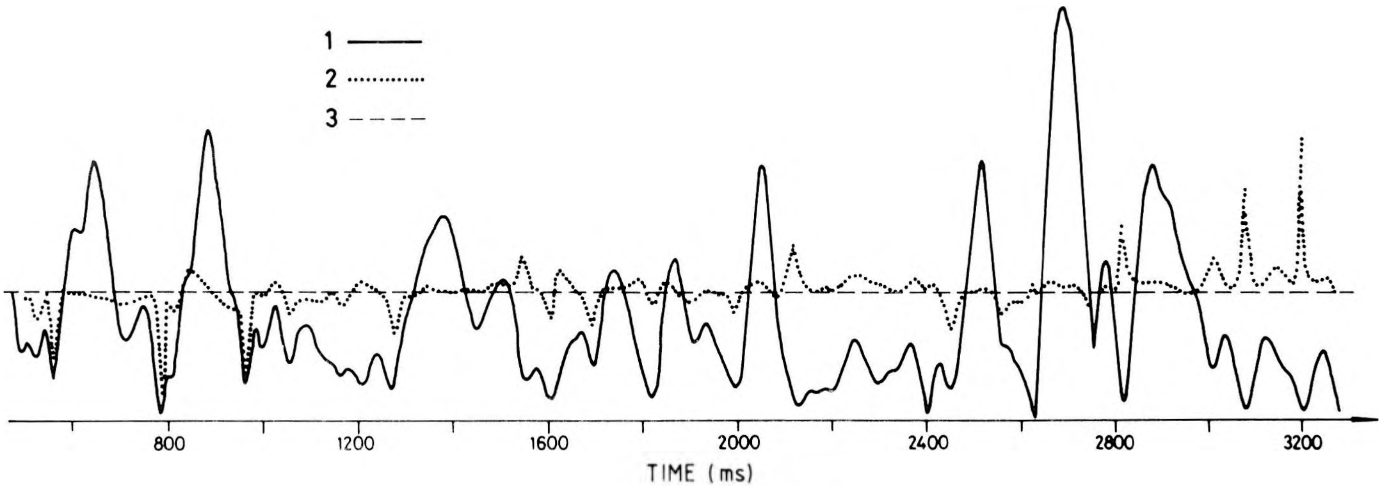

Fig. 2. Envelope and instantaneous frequency plot of a seismic trace
1—envelope; 2—instantaneous frequency; 3—mean Fourier frequency (20 Hz)

2. ábra. Egy szeizmikus jel burkolója és pillanatnyi frekvenciája
1—burkoló; 2—pillanatnyi frekvencia; 3—közepes Fourier frekvencia (20 Hz)

Рис. 2. Огибающая и диаграмма мгновенных частот сейсмической трассы
1—огибающая; 2—мгновенная частота; 3—средняя частота; Фурие (20 Гц)

these two mutually exclusive parameters. Be whatever it may, the existence of signal energy distribution in frequency and time is intuitively correct and a mathematical description of such a phenomenon was introduced by GABOR [1964]. Following RIHACZEK [1968] we define the signal energy density function in t and f as the two-dimensional Fourier transform of the combined autocorrelation function in time and frequency. This is similar to the auto-ambiguity function in Radar theory [VAKMAN 1964]. We take the symmetrical version of the auto-ambiguity function and the corresponding energy density function is:

$$p(t,\omega) = \int \Psi\left(t+\frac{\tau}{2}\right)\Psi^*\left(t-\frac{\tau}{2}\right)e^{-i\omega\tau}d\tau \qquad (18)$$

The signal $\Psi(t)$ is given by equation (2a). $p(t, \omega)$ satisfies the following conditions:

$$\int p(t,\omega)d\omega = |\Psi(t)|^2 \qquad (19a)$$

$$\int p(t,\omega)dt = |\Psi(\omega)|^2 \qquad (19b)$$

$$\int p(t,\omega)dt d\omega = 2E, \qquad (20)$$

E is the energy of the signal.
In can be proved that the first order conditional moment

$$\bar{\bar{\omega}} = \frac{\int \omega p(t,\omega)d\omega}{\int p(t,\omega)d\omega} = \frac{d\Theta(t)}{dt} = \omega(t) \qquad (21a)$$

The variance of $p(t,\omega)$ at a particular instant

$$\bar{\bar{\omega}}^2 - (\bar{\bar{\omega}})^2 = -\frac{1}{2}\frac{\partial^2}{\partial t^2}[\log A(t)] \qquad (21b)$$

From (21a) we infer that the IF provides a measure of Fourier frequency at which the energy/power of a signal acts at an instant of time t [ACKROYD 1970]. It can also be shown that the second conditional moment $\bar{\bar{\omega}}^2$ is related by:

$$\int \bar{\bar{\omega}}^2 A^2(t)dt = \int \frac{d\Psi}{dt}\cdot\frac{d\Psi^*}{dt}dt \qquad (22a)$$

For a narrow band signal the right hand side is by definiton the mean square Fourier frequency [GABOR 1946]. So,

$$\int \bar{\bar{\omega}}^2 A^2(t)\mathrm{d}t = \int \omega^2(t) A^2(t)\mathrm{d}t = \omega^2 \tag{22b}$$

The appropriate weighting factor $A^2(t)$ is important. For Gaussian signals the mathematical expectation of IF may be infinite [STRÖM 1977] if the weight factor is not taken into account. Equation (18) for $p(t,\omega)$ is similar to Wigner's distribution function in quantum mechanics. The well known Fourier inequality $\Delta\omega\Delta t \geq \frac{1}{2}$ follows directly from equation (18) [MOYAL 1949]. By analogy with a piece of music, De Bruijn has termed the function $p(t,\omega)$ as the musical score or simply the score [DE BRUIJN 1967]. We recall that the instantaneous spectrum can be defined from the decompostion of the energy of singnals onto $t - f$ planes [LEVIN 1964]. Although $p(t,\omega)$ is real, one cannot define a physically realizable spectrum because $p(t,\omega)$ is not ≥ 0 for all t. For example, if $t\Psi(t) > 0$ for all $t, p(t,\omega)$ is negative. De Bruijn has shown that certain moving averages yield positive values and the local smoothing (double convolution) of the instantaneous spectrum of a process with the instantaneous spectrum of window function yields the physical spectrum of the process [MARK 1976]. The window function should be such that it is non-negative over $0 < t < \infty$ and its integral over the semiinfinite time domain is unity. We will not enter into details of spectral representation of non-stationary processes in the present article. The above points are mentioned here to stress the fact that a concept of signal energy density in time and frequency leads us not only to the concept of IF for narrow band signals, but with modifications the formulation can be used in defining the short period spectrum of any process. It is however quite complicated to generate a time-dependent spectrum as it is a function of two variables f and t.

6. Spectral dynamics of a seismogram

Classical spectrum analysis is based on the concept of the linear stationary model (at least to the order two). For non-stationary processes the covariance kernel in the Wiener—Khintchine integral is not independent of the time origin. Loynes [1968] while listing the desirable properties of the spectrum of any stochastic process shows that spectral characteristics in the Fourier sense do not exist for non-stationary processes. Other statisticians, notably PRIESTLEY [1965], and MARK [1976] do not agree with such a conclusion and are of the opinion that a local power-frequency distribution at each instant of time exists. From our basic assumption a reflection seismogram is a stationary stochastic process with a continuous spectrum. The sharp peaks and notches observed within a finite time window are attributed to layer thicknesses present within the window. From spectral studies of reflection seismograms it is however very difficult to make out whether the spectral peaks are separate harmonic quasi-harmonic oscillations or are simply narrow peaks of a continuous spectrum. For processes with so-called mixed spectra we have discrete harmonics with a continuous coloured spectrum. Such a process is rarely stationary. If separate narrow band signals are superposed on a process with continuous spectra, due to the additive property of spectra it may appear similar to the spectrum of a reflection seismogram. This is always a non-stationary process. Further if we agree that there is a change of apparent periods of oscillations with time in a seismogram, the process is bound to be non-stationary. Let us consider a purely amplitude modulated process:

$$p(t) = A(t) \cos \omega_0 t$$

ω_0 is the peak Fourier frequency. The deterministic version of such a type of oscillation, known as a Berlage pulse, is given by:

$$f(t) = t^n e^{-\beta t} \sin \omega_0 t$$

n is any integer and β is a constant. Such analytical forms have been used in practice, particularly in simulation of earthquake coda [Farnback 1975]. In seismic exploration $\beta \approx \frac{\omega_0}{\Delta}$. These are narrow band signals and from classical Fourier theory the spectra should contain an infinite number of frequencies. *Figure 3* shows the envelopes and spectra of such pulses. Following Priestley [1981] we can represent the vibration as of only two frequencies ω_0 and $-\omega_0$ each component having a time varying amplitude $A(t)$. Now if the envelope $A(t)$ and the oscillatory part cos ω_0 t are spectrally disjoint, from the product theorem of Bedrosian, the Hilbert transform in given by $A(t) \sin \omega_0 t$. This is true if the envelope is slowly varying with time and then the instantaneous phase is given by

$$\Theta(t) = \arctan \left[\frac{\mathrm{H}\{p(t)\}}{p(t)} \right] = \omega_0 t$$

or IF is equal to ω_0; H denotes the Hilbert transform. Although the Hilbert operator is noncausal and equation (4) is computed by considering the integral over the entire range form $-\infty$ to $+\infty$, the value of $\mathrm{H}\{p(t)\}$ depends very little on the character of random process beyond the duration of the quasiharmonic signal being detected and analysed [Kharchenko et al. 1973]. Such vibrations can be identified on a time scale where $\Theta(t)$ is linear or nearly linear. In IF displays we can detect and locate these frequencies where the IF is nearly constant, preferably for a period of time of the order of $f_0 T \approx 1$. Thus a study of instantaneous frequencies may be helpful to ascertain the dynamic change of frequencies (at least at certain discrete intervals of time) irrespective of the

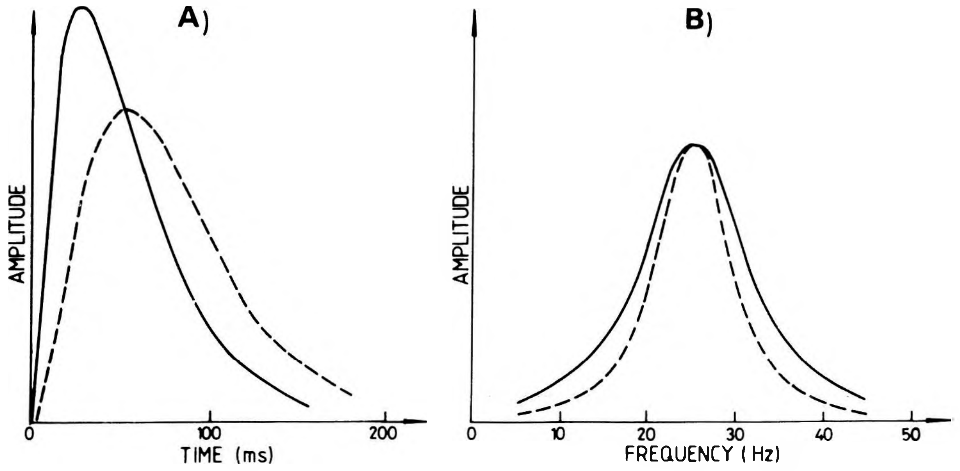

Fig. 3. Envelope (A) and spectrum (B) of Berlage pulse with peak frequency 25 Hz

3. ábra. Egy 25 Hz-es csúcsfrekvenciájú Berlage impulzus burkolója (A), és spektruma (B)

Рис. 3. Огибающая (А) и спектр (В) импульса Берлаж с пиковой частотой 25 Гц

fact that the random realization of a time series belongs to the non-stationary class. *Figures 4a* and *4c* show IF plots of a seismic trace for two different time windows. The trace is from a stacked section of land seismic data. The field seismograms (recording filter 8 Hz—124 Hz) after gain recovery have been stacked with appropriate spherical divergence and NMO corrections. No deconvolution of any kind has been applied. A band pass filter (8 Hz—60 Hz) has been used before obtaining the stacked output. The figures show the presence of two quasi-harmonics with frequencies 22 Hz and 27 Hz at about 1976 ms — 2021 ms and 2532 ms — 2592 ms respectively. Power spectral estimates by three different techniques (FFT, MLM and MEM) of the original seismic trace for the same time windows are shown in *Figures 4b* and *4d.* It is however observed that peak frequency values determined from IF plotes are nearer to peak values given by FFT or the Maximum Likelihood Method than values obtained from the Maximum Entropy Method. The peak freqency at a deeper level (Figure 4d) may sometimes be more than the peak frequency at a shallower level. This has been observed in many areas and was also mentioned earlier by other authors [Taner et al. 1979].

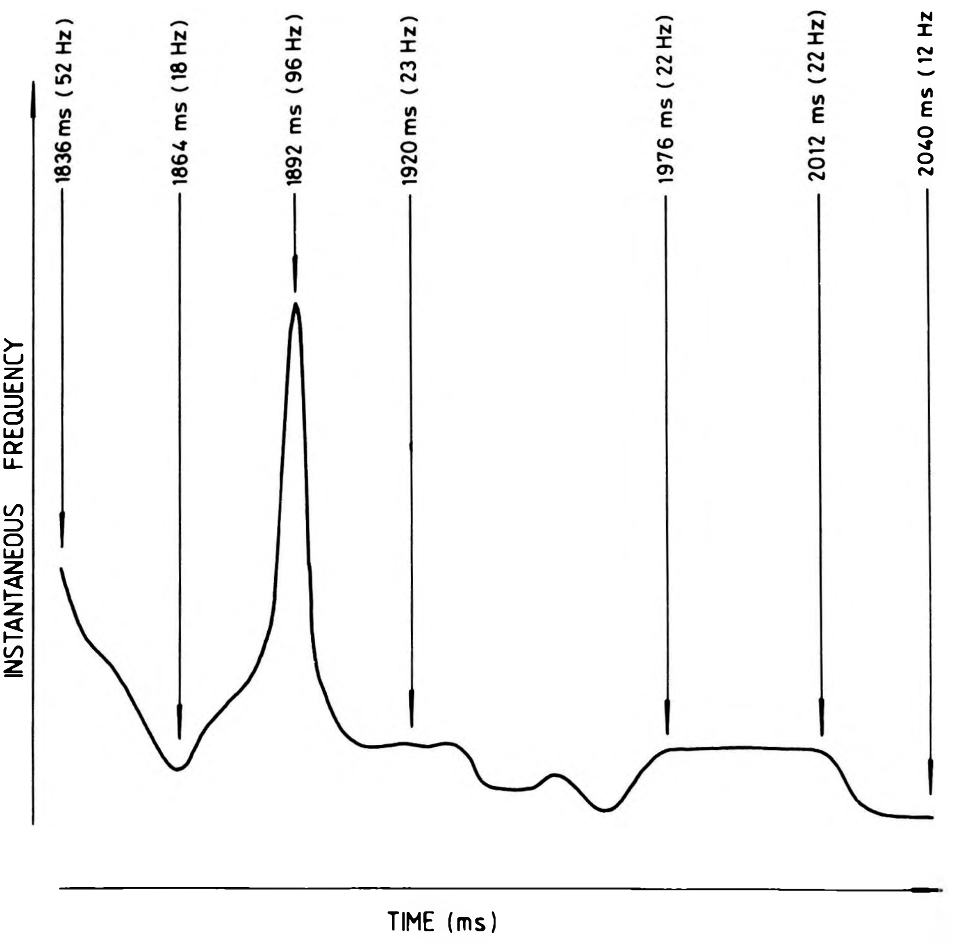

Fig. 4a. Instantaneous frequency plot of a seismic trace: time window 1836 ms—2040 ms

4a ábra. Egy szeizmogram szakasz pillanatnyi frekvencia függvénye. Időablak: 1836 ms—2040 ms

Рис. 4a. Диаграмма мгновенных частот по сейсмической трассе: временное окно 1836 мс—2040 мс

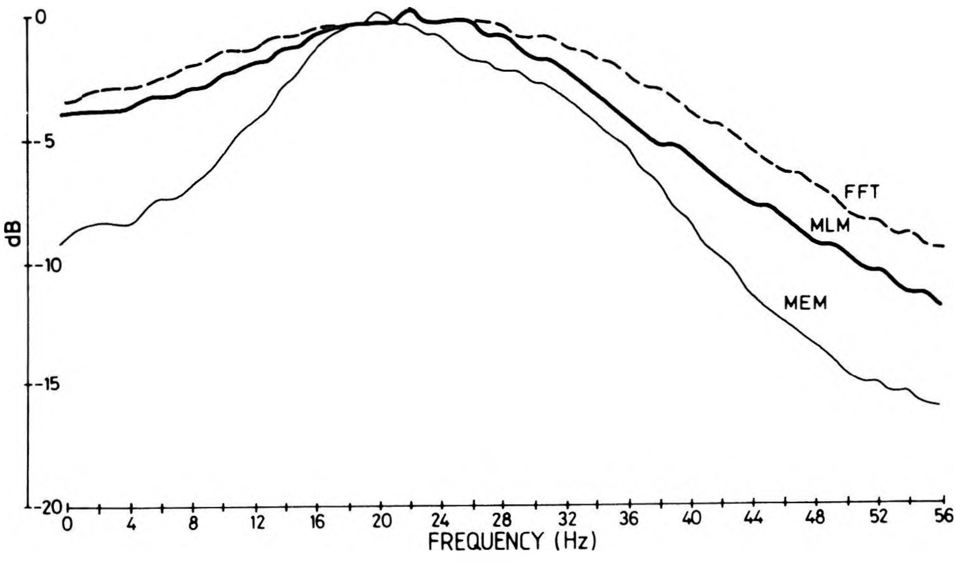

Fig. 4b. Spectral power estimate of a segment of seismic trace: time window 1976 ms—2012 ms

4b ábra. Egy szeizmogram szakasz spektrális energiaeloszlása. Időablak: 1976 ms—2012 ms

Рис. 4b. Оценка спектральой мощности по сегменту сейсмической трассы: временное окно 1976 мс—2012 мс

7. Conclusions

1. The Fourier frequency and instantaneous frequency are fundamentally different quantitites. However for a narrow band signal the variance of IF with respect to mean Fourier frequency is approximately equal to the band-width of the signal. This relation is true for stationary stochastic processes also. One should be cautious to use term by term IF values for the usual frequency interpretation of seismic section.
2. The complex envelope of a seismic envelope of a seismic trace can be determined uniquely by solving the appropriate Sturm—Liouville differential equation. The "instantaneous frequency" may often be strongly coupled with the envelope function. The location of maxima/minima of envelope (signal strength) can be predicted with fair accuracy from IF displays. The IF display may be helpful for seismic correlations where signal strengths are less.

3. An attempt has been made in this paper to "embrace" the two mutually exclusive parameters, Fourier frequency and IF, by utilizing the concept of signal energy density function in time and frequency. It is shown that IF is a measure of Fourier frequency at which the signal energy density is concontrated at a particular instant of time. The interpretation is meaningful if the energy density function is real and positive. This also explains the occasionally observed phenomenon where most of the signal energies are transmitted by filters with band-widths much narrower than the signal band-widths. The expression for energy density function considered in this paper, when suitably modified, can also be used to define a physically realizable short-time spectrum of any process (stationary or non-stationary).

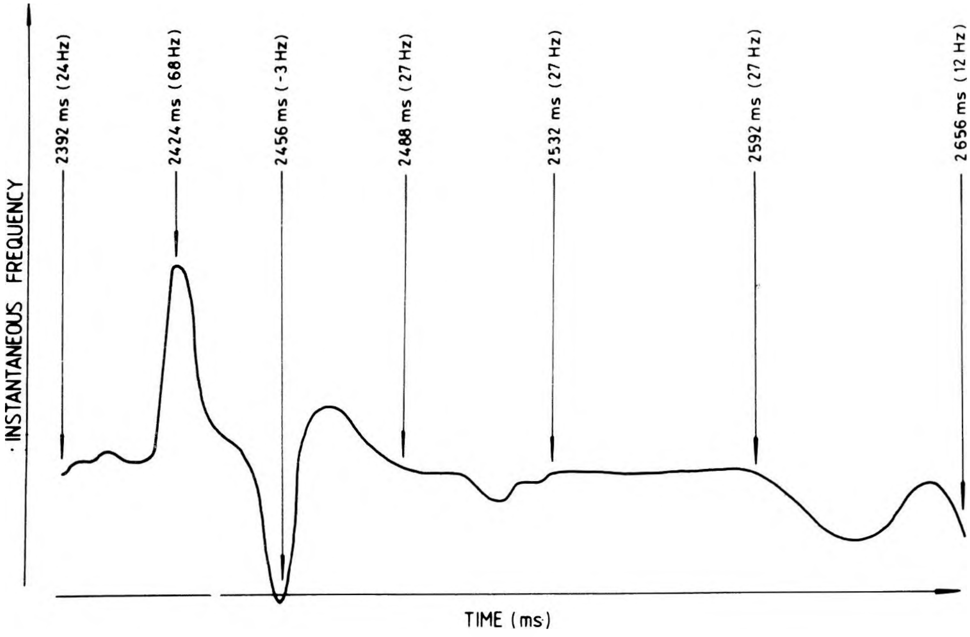

Fig. 4c. Instantaneous frequency plot of the same seismic trace as in Figure 4a; time window 2392 ms—2656 ms

4c ábra. A 4a ábrán szereplő szeizmogram pillanatnyi frekvencia függvénye a 2392 ms—2656 ms időablakban

Рис. 4c. Диаграммы мгновенных частот по сейсмической трассе, показанной на Рис. 4a: временное окно 2392 мс—2656 мс

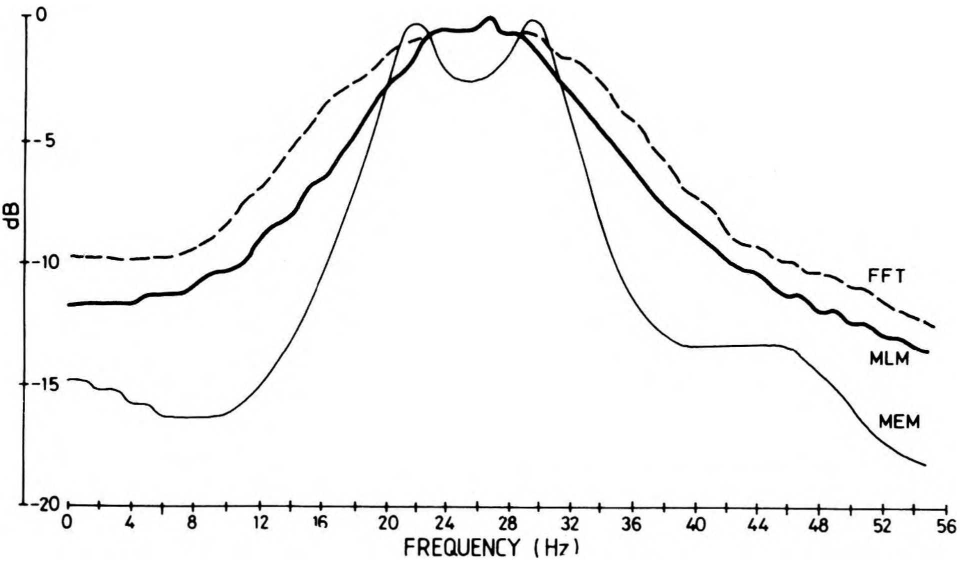

Fig. 4d. Spectral power estimate of the same seismic trace as in Figure 4a): time window 2532 ms—2592 ms

4d ábra. A 4a ábrán szereplő szeizmogram spektrális energiaeloszlása a 2532 ms—2592 ms időablakban

Рис. 4d. Оценка спектральной мощности по сеймической трассе, показанной на рис. 4а: временное окно 2532 мс—2592 мс.

4. The analytic signal technique can be exploited for decompostion of any random process into a number of quasi-harmonic oscillations at certain discrete time intervals of reflection seimograms. The peak Fourier frequencies determined from IF desplays may be used as additional information for ascertaining transition zones in a seismic section. The information obtained by this technique does not depend on the overall characteristics of the process.

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A REFLEXIÓS SZEIZMOGRAMOK SPEKTRÁLIS SZERKEZETÉNEK TANULMÁNYOZÁSA A PILLANATNYI FREKVENCIA VIZSGÁLATA ALAPJÁN

J. G. SAHA

A szeizmogramok spektrális jellemzői gyakran emlékeztetnek a kevert spektrumú folyamatok jellemzőire. Az ilyen folyamatok rendszerint nemstacionáriusak. A jelanalízis módszerével előállítható pillanatnyi fázis – pillanatnyi frekvencia függvények lehetővé teszik a szeizmogram kváziharmonikus tartalmának kimutatását. A módszer a folyamat stacionárius vagy nemstacionárius voltára való tekintet nélkül érvényes. A kváziharmonikusok kifejlődési és lecsengési folyamatai is kellő pontossággal követhetők a pillanatnyi frekvencia ismeretében.

СПЕКТРАЛЬНАЯ СТРУКТУРА СЕЙСМОГРАММ МОВ, ПОЛУЧЕННЫХ В РЕПРЕЗЕНТАЦИИ ПО МГНОВЕННОЙ ЧАСТОТЕ

Й. Г. САХА

Спектральная характеристика сейсмограмм часто похожа на процессы со смешанными спектрами. Фазовые/мгновенные частоты, выведенные путем техники анализа из сигналов, позволяют обнаружить присутствующие в сейсмограмме квази-гармонические частоты. Метод может найти применение независимо от того, носит ли процесс установившийся характер или нет. Временная эволюция и затухание таких квази-гармонических частот могут быть оценены с достаточной точностью по мгновенной визуализации частоты.