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POSSIBLE VARIATIONS OF THE MOMENTUM OF INERTIA AND OF THE ELLIPTICITY OF THE EARTH DURING THE LAST FIVE HUNDRED MILLION YEARS

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Secular variations in the rate of rotation of the Earth are mainly due to the effect of the Moon and the Sun. The tidal bulge brought about by these celestial bodies decelerates the rotation of our planet around its axis in case of the solid Earth and the oceans, while atmospheric tide has an accelerating effect. This latter phenomenon is only partially of gravitational origin, the main factor causing it is the semi-diurnal thermal wave aroused by the Sun, whose effect is greatly magnified by the characteristic vibration of near 12 hrs. period of the Earth's atmosphere. The atmospheric semi-diurnal tide is of positive phase-lag, i.e. it brings about an accelerating moment. According to our recent knowledge the variations of rate of rotation of the Earth around its axis are mostly due to oceanic tides. The tide of the solid Earth plays only a subordinate role, for the phase differences determined at the majority of stations recording tidal phenomena usually do not exceed —0,°4, which corresponds to a delay of appr. 1,°0 of the tidal bulge.

Oceanic tides cannot be investigated without reliable cotidal maps. Since up to now no such maps have been available, the effect of the seas could not have been exactly accounted for.

According to the principle of conversation of momentum:

$$I \cdot \omega + \frac{M \cdot m}{M + m} \cdot c^2 \cdot n + \frac{(M + m)m_\odot}{M + m + m_\odot} c^2_\odot n_\odot = \text{const.} \tag{1}$$

In Eq. (1):

I = the polar momentum of inertia of the Earth,
ω = velocity of rotation of the Earth,
$n, n_\odot$ = orbital velocity of the Moon and Sun, resp.,
$M, m, m_\odot$ = mass of the Earth, Moon and Sun, resp.,
c = Earth–Moon distance,
$c_\odot$ = Earth–Sun distance.

Applying Kepler's law for the Moon:

$$n^2c^3 = f(M + m),$$

f being the gravitational constant. Denoting the Ephemeris Time by t:

$$\frac{dc}{dt} = -\frac{2}{3}\frac{c}{n}\frac{dn}{dt} \tag{2}$$

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and, because of

$$m \cdot n^2 \cdot \left(c \frac{M}{M+m}\right) = f \frac{mM}{c^2},$$

substituting Eq. (2) into Eq. (1) yields (if M, m and $m_{☉}$ are constants):

$$\frac{d(I \cdot \omega)}{dt} = \frac{1}{3}\left(\frac{m \cdot M}{M+m} \cdot c^2 + \frac{m_{☉}(M+m)}{M+m+m_{☉}} \cdot c^2_{☉}\right) = N_{☾}\left(1 + \frac{N_{☉}}{N_{☾}}\right), \quad (3)$$

where N denotes momenta of the Moon and Sun, respectively. From astronomical data

$$\frac{d\omega}{dt} = -5{,}6 \cdot 10^{-22}\ \mathrm{sec}^{-2}.$$

Assuming that I does not depend on time, we obtain

$$I \frac{d\omega}{dt} = -4{,}49 \cdot 10^{23} \frac{\mathrm{g} \cdot \mathrm{cm}^2}{\mathrm{sec}^2}.$$

Various authors have analyzed the growth rings of corals from different geological epochs and from that determined the former values of the velocity of rotation of the Earth. These findings are compiled in Table I, after a collection of papers ed. by RUNCORN (1970). It should be noted that the length of the year determined for the Devon has an error of appr. ±5–7 days.

According to this Table I it can be stated that $\frac{d\omega}{dt}$ have had a value similar to its present one during the past five hundred million years.

Table I
I. táblázat
Таблица I

Geological Epoch Geológiai kor Геологический возраст	Absolute Age 10^9 years Abszolút kor 10^9 év Абсолютный возраст 10^9 лет	Length of Day hours A nap hossza órákban Продолжительность сутки в часах	Length of Year days Év hossza napokban Продолжительность года в днях	$\frac{d\omega}{dt} \cdot 10^{-22}\ \mathrm{sec}^{-2}$	Author (cited by RUNCORN, 1970) Szerző (*Runcorn* 1970 cikkgyűjtemény alapján) Авторы (по Ранкорнуе 1970)
Ordovician Ordovicium Ордовик	0,45	21,2	413,2	—6,78	Wells
Devonian Devon Девон	0,39	21,9	400,0	—5,68	Wells
Devonian Devon Девон	0,39	21,2	413,2	—7,82	Scrutton
Carboniferous Carbon Карбон	0,36	22,1	396,4	—5,52	Wells
Jurassic Jura Юра	0,09	23,5	372,8	—5,46	Barker

Using different cotidal maps KUZNETSOV (1972) succeeded in determining the values of $N_{\leftmoon} + N_{\odot}$ and $N_{\odot}/N_{\leftmoon}$. His results, which take into account the tide of the solid Earth as well, are as follows:

$$N_{\leftmoon} + N_{\odot} = -7{,}0 \cdot 10^{23} \frac{\text{gcm}^2}{\text{sec}^2}\ ;\ N_{\leftmoon}/N_{\odot} \approx 7{,}0 - 7{,}4.$$

From this, the Earth's tide is $-1{,}33 \cdot 10^{23} \frac{\text{gcm}^2}{\text{sec}^2}$. In what follows we will neglect the accelerating momentum of atmospheric tide which is appr. $+0{,}37 \cdot 10^{23} \frac{\text{gcm}^2}{\text{sec}^2}$ (HOLMBERG, 1952).

Using KUZNETSOV's data and assuming that $I =$ const., we obtain from Eq. (3)

$$\frac{d\omega}{dt} = -8{,}6 \cdot 10^{-22}\ \text{sec}^{-2}$$

This value, however, does not agree with either recent or paleontological data. In order to set up again the balance of Eq. (3), we either have to assume some factor which would accelerate the rotation of the Earth at a rate of $2{,}5 \cdot 10^{-22}\ \text{sec}^{-2}$, or the assumption about the independence on time of the momentum of inertia should be rejected. Since we do not know of any factor—apart from the atmosphere—accelerating the rotation of the Earth around its axis, the hypothesis of a constant momentum of inertia has to be ruled out. In this case, from Eq. (3)

$$\frac{dI}{dt} = \left| N_{\leftmoon}\left(1 + \frac{N_{\odot}}{N_{\leftmoon}}\right) - I\frac{d\omega}{dt} \right| \cdot \omega^{-1} = -3.25 \cdot 10^{27} \frac{\text{gcm}^2}{\text{sec}},$$

implying that $0{,}45 \cdot 10^9$ years ago (Ordovician) the momentum of inertia of the Earth had been $8{,}5 \cdot 10^{44}\ \text{gcm}^2$, as against its present value of $8{,}024 \cdot 10^{44}$. Obviously the value of $\frac{dI}{dt}$ could not have been such large throughout the $5{,}0 \cdot 10^9$ years history of the Earth, since in that case some $(1{,}7 - 1{,}8) \cdot 10^9$ years ago our planet would have reached the state of instability. Indeed, for that time the value of I would be $9.7 \cdot 10^{44}\ \text{gcm}^2$ which corresponds to the momentum of inertia of the homogeneous Earth.

Such a significant change of the momentum of inertia of the Earth during the past five hundred million years certainly must have been associated with a radial rearrangement of masses. In what follows we will try to estimate the degree of this rearrangement. Let us recall first of all Legendre's equation which assumes that our planet is, in its entirety, hydrostatically balanced.

According to this equation:

$$\frac{d\varrho(r)}{dr} = -\frac{\beta^2}{4\pi f} g, \tag{4}$$

where $\varrho(r)$ is density in function of the Earth's radius ($0 \leq r \leq a$, $a = 6371$ km), g is gravitational acceleration, β is a constant of proportionality: $\beta = \beta(I, M, a)$. Equation (4) yields, upon integration, the function $\varrho(r)$ independently of the elastic parameters $\lambda(r)$ and $\mu(r)$. Since we don't have any information on the value of these parameters in the past geological ages we have found appropriate to use Legendre's equation even though we realize that the $\varrho(r)$ function obtained might differ from the real density distribution. It is hoped that a comparison of models constructed by Eq. (4) using recent and past data would yield an estimate of the order of magnitude of the changes having occured during the evolution of the Earth.

In our computations we started out from the recent and the Ordovician values of the momentum of inertia and determined the corresponding density functions $\varrho(r)$ (Fig. 1). It has been assumed that the radius (a) and mass (M) of the Earth had not changed significantly in this period. As it is evident from the figure, the near-surface value of density has decreased during the past five hundred million years, materials of greater density have moved towards the centre of the Earth.

For a more detailed scrutiny of the possible variations of $\varrho(r)$, some further assumptions have to be made. On the basis of the assumptions made above (about M and a being constant) it is reasonable to suppose that the density $\varrho(0)$ in the centre of the Earth practically has not changed during the past $0,5 \cdot 10^9$ years. Possible variations of the elastic parameter $\lambda(r)$ do not affect significantly the results of the computations, so a recent $\lambda(r)$ function can be used for models of $0,5 \cdot 10^8$ years ago as well.

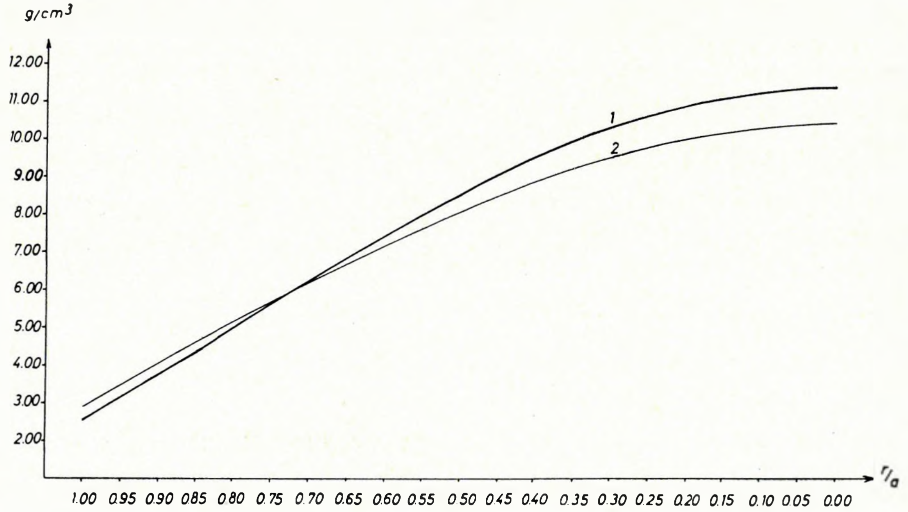

Fig. 1 Density distribution on the Earth, recently *(1)* and $0,5 \cdot 10^9$ years ago *(2)* according to Legendre's equation ($a = 6371$ km)

1. ábra. A föld sűrűségeloszlása jelenleg *(1)* és $0,5 \cdot 10^9$ évvel ezelőtt *(2)*, Legendre képlete szerint ($a = 6371$ km)

Рис. 1. Распределение плотности Земли, в настоящее время (1) и $0.5 \cdot 10^9$ лет тому назад (2) по формуле Лежандра ($a = 6371$ км)

The function $\varrho(r)$ can be found from the following system of equations:

$$\frac{dM(r)}{dr} = 4\pi\varrho(r) \cdot r^2, \tag{5}$$

$$\frac{dI(r)}{dr} = \frac{8\pi}{3}\varrho(r) \cdot r^4, \tag{6}$$

$$\frac{d\varrho(r)}{dr} = \frac{g(r) \cdot \varrho(r)}{\Phi(r)} \cdot \varkappa(r), \tag{7}$$

where $\varkappa(r)$ is a function characterizing the inhomogeneity; $\Phi(r) = \frac{K(r)}{\varrho(r)}$; $K(r) = \lambda(r) + \frac{2}{3}\mu(r)$. Using the expression $\varkappa(r)$ given by MOLODENSKY (1953):

$$\varkappa(r) = \frac{K(r)}{\lambda(r)},$$

Eq. (7) yields

$$\frac{d\varrho(r)}{dr} = -\varrho^2(r) \cdot g(r) \cdot \lambda^{-1}(r) \tag{8}$$

The solution of this problem would generally require the knowledge of the values of $I(a)$, $M(a)$ and $\varrho(a)$. In the present case, however, we do not know the value of $\varrho(a)$, and the exact position of the core-mantle boundary of $0{,}5 \cdot 10^9$ years ago is also unknown. Let us try to determine these unknown quantities from the above system of differential equations, making use of the conditions $\varrho(0) = \text{const}$, $M = \text{const}$, $\frac{d\lambda(r)}{dt} = 0$. Introducing the function

$$y(r) = \frac{4\pi f}{r^3} \int_0^r \varrho(r) \cdot r^2\, dr,$$

we have

$$\varrho(r) = \frac{1}{3}\left[r\frac{dy(r)}{dr} + 3y(r)\right],$$

$$g(r) = r \cdot y(r)$$

and Eq. (8) becomes

$$\frac{d^2y(r)}{dr^2} + \frac{4}{r}\frac{dy(r)}{dr} + y(r) \cdot \left[r\frac{dy(r)}{dr} + 3y(r)\right]\frac{\varrho(r)}{\lambda(r)} = 0, \tag{9}$$

because of the continuity of $g(r)$ the function $y(r)$ is continuous at the core-mantle boundary and—for $y(o)$ has to be finite—at the centre of the Earth $\frac{dy(o)}{dr} = 0$.

At the Earth's surface obviously $y(a) = a \cdot g(a)$. Making use of these properties of the function $y(r)$ we have succeeded to determine $\varrho(a)$ and the position of the core-mantle boundary for $0{,}5 \cdot 10^9$ years ago. Using a similar model and recent values of $M(a)$, $I(a)$ and $\varrho(a)$ ($= 3{,}34$ gcm^{-3}) Molodensky (*op. cit.)* obtained:

$$\varrho(0) = 12{,}58 \text{ gcm}^{-3}$$

core-mantle boundary = 0,55 Earth radius.

The solution of Eq. (9) is rendered difficult by the fact that $\frac{dy}{dr}$ as well as ϱ/λ is discontinuous at the core-mantle boundary. To overcome this difficulty, computations has been performed as follows:

1. We solved numerically the differential equation (9), starting out from the centre of the Earth. Since the position of the core-mantle boundary is unknown, the computations were first performed up to the spherical boundaries $r/a = 0{,}60$; 0,55; 0,50, respectively.

2. Since $\frac{dy}{dr}$ is discontinuous at the boundary, we assumed the values $\frac{dy}{dr} =$ $= -5{,}0$; $-5{,}5$; $-6{,}0$; -6.5 and continued the integration of Eq. (9) up to the surface, making use of the respective values. Thus, we obtained 12 curves altogether which were then used to construct by means of interpolation the solution satisfying Eq. (6).

According to these computations, some $0{,}5 \cdot 10^9$ years ago, in the Ordovician, the near-surface density was 3,6 g/cm^3, the core-mantle boundary being at 0,58 r/a.

Such significant variations of the angular velocity of the Earth should have changed the ellipticity of the Earth as well. Let us characterize the deformation of the Earth by the so-called secular Love number (Munk, MacDonald, 1960):

$$K_s = \frac{3fHI}{\alpha^5 \omega^2},$$

H being the dynamic ellipticity of the Earth. The ellipticity α deviates somewhat from that valid to a hydrostatically balanced Earth, but this deviation should not be considered as significant in the present case. So, our planet can be considered as a hydrostatical figure ($\alpha = 0{,}00335$).
Consequently

$$I = \frac{2}{3} \frac{Ma^2}{H} \left(\alpha - \frac{q}{2}\right); \quad q = \frac{\omega^2 a^3}{f \cdot M},$$

i.e.

$$K_s = \frac{2fM\alpha}{a^3\omega^2} - 1. \tag{10}$$

Using recent data, $K_s = 0{,}94$. Since the secular Love number describes the state of the Earth under the effect of forces of very long duration, it can be assumed that K_s had been constant in the different geological epochs. Assuming again the constancy of the Earth's radius and taking the value of $\frac{d\omega}{dt}$ as constant, according to

Table I, then:

$$\frac{\alpha_0}{\omega_0^2} = \frac{\alpha_p}{\omega_p^2},$$

i.e.

$$\alpha_p = \alpha_0 \left(\omega_0 + \Delta t \frac{d\omega}{dt}\right)^2 \omega^{-2} \tag{11}$$

where index $_0$ refers to recent values, index p to values having been valid $0{,}5 \cdot 10^9$ years ago, of the ellipticity and angular velocity. Carrying out the computations, Eq. (11) yields $\alpha_p = 0{,}00433$.

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VARGA PÉTER

A FÖLD INERCIAMOMENTUMÁNAK ÉS LAPULTSÁGÁNAK LEHETSÉGES VÁLTOZÁSAI AZ UTOLSÓ FÉLMILLIÁRD ÉV ALATT

A Föld csillagászati adatokból meghatározott szekuláris forgássebesség változása ($-5{,}6 \cdot 10^{-22}$ sec^{-2}) és ezen mennyiség különböző földtörténeti korokban élt korallok növekedési gyűrűi alapján meghatározott értéke közelítően egyenlőek. Bolygónk tengely körüli forgásának lassulását elsősorban a tengerek árapálya okozza (a szilárd Föld és az atmoszféra árapálya alárendelt szerepet játszanak). A világóceánok kotidális térképeinek felhasználásával megállapítható, hogy ha a Föld poláris inerciamomentumának mai értékével számolunk ($8{,}024 \cdot 10^{44}$ gcm^2) $8{,}59 \cdot 10^{-22}$ sec^2 szekuláris lassulás adódik. Mivel a Föld forgását $3{,}0 \cdot 10^{-22}$-kel gyorsító hatót jelenleg sem bolygónk belsejében, sem környezetében nem ismerünk, az adódó ellentmondást úgy tudjuk csak feloldani, ha az inerciamomentumot nem tekintjük állandónak. Ebben az esetben $-3{,}25 \cdot 10^{27}$ gcm^2 momentumváltozás érték adódik, amelynek hatására a Föld belsejében radiális tömegátrendeződések játszódtak le az Ordoviciumtól napjainkig. Ez a momentum változás érték azonban nem lehetett érvényes a Föld egész története folyamán, mert hatására az inerciamomentum értéke $1{,}7 \cdot 10^9$ évvel ezelőtt $9{,}7 \cdot 10^{44}$ gcm^2 értékre kellett volna hogy növekedjék, azaz ennél az időpontnál régebben bolygónk instabil felépítését kellene feltételeznünk.

П. ВАРГА

ВОЗМОЖНЫЕ ВАРИАЦИИ МОМЕНТА ИНЕРЦИИ И СЖАТИЯ ЗЕМЛИ ЗА ПОСЛЕДНИЕ ПОЛМИЛЛИАРДА ЛЕТ

Вековые вариации скорости вращения Земли, определенные по астрономическим данным ($-5,6 \cdot 10^{-22}$ сек$^{-2}$) и величины, определенные по кольцам роста кораллов различных геологических эпох приблизительно совпадают. Замедление вращения нашей планеты вокруг своей оси вызывается прежде всего приливами морей (приливы твердой Земли и атмосферы играют здесь подчиненную роль). С использованием котидальных карт мировых океанов можно делать вывод о том, что если считать с современной величиной момента полярной инерции Земли ($8.024 \cdot 10^{44}$ гсм^2), то получается вековое замедление равное $8,59 \cdot 10^{-22}$ сек^2. Поскольку в настоящее время ни внутри Земли, ни в ее окружности не известны факторы, которые могли бы ускорить вращение Земли на $3.0 \cdot 10^{-22}$, возникающее противоречие можно разрешить только тогда, если момент инерции не считается постоянным. В этом случае получается изменение момента $-3,25 \cdot 10^{27}$ гсм^2/сек, в результате которого в недрах Земли происходило радиальное перераспределение масс от ордовика до настоящего времени. Однако, такое изменение момента не могло действовать за всю историю развития Земли, поскольку в этом случае $1,7 \cdot 10^9$ лет тому назад величина момента инерции должна была бы возрастать до $9,7 \cdot 10^{44}$ гсм^2, т. е. нужно было бы предполагать неустойчивое строение Земли до указанного времени.