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ALGORITHM COMPUTING ELECTROMAGNETIC WAVE SCATTERING FROM ROUGH SURFACES

INTRODUCTION

Mostly, when one is talking about rough surfaces and wave scattering from rough surfaces there is assumed random rough surfaces. Practically it means that we deal with the surface that actual geometrical shape is not defined. Instead of exact forms and scattering properties the average approach to real scattering pattern is considered.

D. E. Barrick defines three basic kinds of problems in his part of RCS handbook, ref. [3], dedicated to rough surfaces. Named without deeper description, there are direct scattering problems, inverse scattering problems and clutter problems. Direct scattering problem is characterised by finding average properties of scattered signal when the surface properties are known. In case of inverse scattering problem one try to obtain statistical information about the rough surface from the characteristics of scattered field. Clutter problem is more less application of direct problem since properties of for instance ground clutters are sought and that information is used to better unwanted clutter suppression. There is clear the direct scattering problem is the problem of cross section prediction techniques and scattered field simulation method.

Important part of the prediction technique when reasonable precision of results is required of course is the equality of statistical parameters of surface model compared to natural surface. The same literature mentions following three types of rough surface models. First is so called semi-empirical model. This group of models for the most part offers the simplest results. All such models involve a set of arbitrary constants that are functions of the properties of the actual surface and are determined by matching the model to measured parameters of real surface. Second kind of model is geometrical model. These models employ a surface made up of deterministic or simple shapes arranged randomly over a planar area. There is example of the geometrical model, the vegetation-like rough surface model consisting of thin dielectric cylinders, arranged randomly but preferring a vertical orientation. Last type of model is statistical model, which treats the surface height above xy-plane as a random variable. That class of models is the most general and provides results explicitly in terms of the average surface properties rather than arbitrary constants. The statistical models are the most suitable for numerical RCS prediction techniques and computer simulation.

ROUGH SURFACE GENERATION

There are several various methods used for random rough surface generation and each of them produces surfaces with specific character. J. A. Oglivy in his book, reference [2], gives more extend overview end description of those methods. In this stage of my work is not so important to have exact model of any natural rough surface, more important, in my opinion, is to have control above statistic parameters of created surface and to evolve their influence into scattering pattern.

The method that gives good possibilities to generate specific surface as a random function of surface height satisfying required statistical distribution, in that case Gaussian distribution, and the method that I used is moving average process.

Theory of the moving average process

Three-dimensional surface characterised by dimensions along axes *x* and *y* and its height is in form $h_{np}=h(n\Delta x, m\Delta y)$ with Δx and Δy discretisation intervals can be generated by:

$$h_{np} = \sum_{l=0}^{N} \sum_{m=0}^{M} w_{lm} \cdot u_{n-l,p-m} \tag{1}$$

where variable *u* is uncorrelated random variable and vector *w* is weight vector, which gives statistical character of created surface, especially shape of the correlation function. Its length *N* then determines the order of moving average process. Used random number generator and its quality has influence over random character of created surface

Weight matrix used to create surface with the Gaussian character of correlation function can have form:

$$w_{lm} = w_0 \cdot \exp\left\{ -2 \cdot \left[\frac{(l\Delta x)^2}{\lambda_x^2} + \frac{(m\Delta y)^2}{\lambda_y^2} \right] \right\} \tag{2}$$

where λ_x, λ_y are correlation lengths in the direction x and y, respectively. Constant w_0 specifies extension of h_{np}.

WAVE SCATTERING COMPUTATION

The total field in presence of scatterer can be written as the sum of incident and scattered field:

$$\varphi(\vec{r}) = \varphi^{inc}(\vec{r}) + \varphi^{sc}(\vec{r}) \tag{3}$$

Then vectors $\boldsymbol{k_{inc}}$ and $\boldsymbol{k_{sc}}$ are the wavevectors of incident and scattered waves. If the wavelengths of those waves are equals and speed of propagation is not changed, wavenumbers are equals only directions are different.

$$\begin{aligned} \vec{k}_{inc} &= k\widehat{R}_{inc} \\ \vec{k}_{sc} &= k\widehat{R}_{sc} \end{aligned} \tag{4}$$

are the definitions of local incident and scattered wavevectors.

Kirchhoff theory

According to Physical Optics or Kirchhoff theory the scattered field can be computed by:

$$\varphi^{sc}(\vec{r}) = 2j\int_S (\vec{n}_0 \cdot \vec{k}_{inc})\varphi^{inc}(\vec{r}_0)\left(\frac{e^{jk|\vec{r}-\vec{r}_0|}}{4\pi|\vec{r}-\vec{r}_0|}\right)dS(\vec{r}_0) \tag{5}$$

Every four adjacent points of the generated surface model form a small planar rectangular patch of dimensions Δx and Δy with the normal vector $\boldsymbol{n}$. Algorithm is based on the idea of summing the scattered field contributions from all N times M patches. If the incident wave is assumed of spherically spreading then equation 5 becomes:

$$\varphi^{sc}(\vec{R}_r) = 2j\int_{S_{n,m}} A(\vec{r}_0)(\vec{n}_0 \cdot \vec{k}_{inc})\left(\frac{e^{jk|\vec{r}_0-\vec{R}_s|}}{4\pi|\vec{r}_0-\vec{R}_s|}\right)\left(\frac{e^{jk|\vec{R}_r-\vec{r}_0|}}{4\pi|\vec{R}_r-\vec{r}_0|}\right)dS_{n,m}(\vec{r}_0) \tag{6}$$

where $S_{n,m}$ is the area of computation of *(n,m)*-th patch at $x_0=n\Delta x$ and $y_0=m\Delta y$. Vectors $\boldsymbol{R}_s$ and $\boldsymbol{R}_r$ are the vectors of source and receiver respectively with direction pointing from the origin.
The amplitude of incident wave $A(\boldsymbol{r}_0)$ is constant over each patch in this applications but generally it is possible to define dependencies between amplitude and antenna pattern, frequency characteristics of the system or absorption characteristics of the surface.

Roughness criterion

After the study of referenced literature, especially reference [1]. I know two approaches to compute electromagnetic field scattered from rough surface. There is one type of methods based on the perturbation theory that are useful for slightly rough surfaces. Next main kind of methods is derived from the Physical Optics and is represented by the Kirchhoff theory. These methods are sufficient mostly for higher degree of roughness than perturbation theory. It is necessary to define a criterion to decide if specified surface is still smooth or if it is rough and to divide slightly rough and rough surfaces.
One possibility is to use Rayleigh criterion, reference [1], chapter 1.2. This criterion is based on the comparison of phase differences between two parallel rays scattered from separate points of surface. Because these two rays are parallel their incident angles are equal $\theta_1=\theta_2$. Because they are scattered from separate points the heights of surface at these points are different $\Delta h=h_1-h_2$. Phase difference can be computed by:

$$\Delta\phi = 2k\cdot\Delta h\cdot\cos\theta_1 \tag{7}$$

The interference of these rays depends on $\Delta\phi$, of course. According to the phase difference we can decide that surface is smooth if $\Delta\phi<\pi/2$, otherwise it is rough. When Rayleigh parameter will be defined:

$$R_a = k\sigma\cdot\cos\theta_1 \tag{8}$$

where Δh can be replaced by RMS deviation of height σ. Then Rayleigh criterion of roughness becomes:

$$R_a < \frac{\pi}{4} \tag{9}$$

At this work I developed algorithms using only the Kichhoff theory that means I am working with surfaces with higher degree of roughness.

Kirchhoff theory application

If the frequencies of incident and scattered waves are equal and environment of propagation is the same for both of these waves wavenumbers $k_{inc}=k_{sc}=k$ hence I can denote:

$$\begin{aligned}\vec{R}_{inc} &= \vec{r}_0 - \vec{R}_s \\ \vec{R}_{sc} &= \vec{R}_r - \vec{r}_0 \\ \vec{k}_{inc} &= k\widehat{R}_{inc}\end{aligned} \qquad (10)$$

Then equation 9 can be rewritten to the form:

$$\begin{aligned}&\varphi^{sc}_{n,m}(\vec{R}_r,\vec{R}_s) = 2jA\cdot\int_{S_{n,m}}(\vec{n}_0\cdot k\widehat{R}_{inc})dS_{n,m}(\vec{r}_0)\cdot \\ &\frac{1}{4\pi}\int_{S_{n,m}}\frac{e^{jkR_{inc}}}{R_{inc}}dS_{n,m}(\vec{r}_0)\cdot\frac{1}{4\pi}\int_{S_{n,m}}\frac{e^{jkR_{sc}}}{R_{sc}}dS_{n,m}(\vec{r}_0)\end{aligned} \qquad (11)$$

The patches in this application are rectangular and planar the first integral – scalar multiplication of normal vector $\boldsymbol{n_0}$ and wavevector of incident wave $\boldsymbol{k_{inc}}$- in equation 3.10 equation becomes equal to scalar multiplication of the normal vector $\boldsymbol{n_0}$ and the wavevector in the midpoint of the patch $\boldsymbol{k_{inc\,MID}}$.

$$\vec{k}_{incMID} = k\hat{R}_{incMID} \qquad (12)$$

Then we have to solve two remaining integrals of the Green's function.
Principe of numerical solution is to divide rectangular patch into the sufficient number of rectangular sub-patches of dimension Δx_{sub} and Δy_{sub} for that it is possible to consider spherically spreading function as constant and to write for the *(u,v)*-th patch:

$$\varphi^{GR}(R) = \frac{1}{4\pi}\int_{S_{U,V}}\frac{e^{jkR}}{R}dS_{u,v}(\vec{r}_0) = \Delta x_{sub}\Delta y_{sub}\frac{e^{jkR}}{4\pi R} + C_{err} \qquad (13)$$

The error constant C_{err} has to be smaller than tolerance limit defined on the beginning of the computation. The numerical integration was launched as the iteration process when each iteration adds one row and column of sub-patches and instead of real error in sense of difference between planar and spherical spreading there is computed estimation of this error as the difference between two following iterations:

$$C_{est} = \varphi_i^{GR} - \varphi_{i-1}^{GR} \tag{14}$$

The *U* and *V* are numbers of sub-patches in the *y* and *x* direction of the *(n,m)*-th patch. This algorithm works with the equal number of sub-patches in *x* and *y* direction.

$$\begin{aligned} \Delta x &= U \cdot \Delta x_{sub} \\ \Delta y &= V \cdot \Delta y_{sub} \end{aligned} \tag{15}$$

Number of iterations is there denoted as *i*.
Practically it is in deed too long computation time to solve this integral by iteration process and to evaluate the same patch several times to fit the tolerance condition. Better solution is to find dependency between the final error estimation and number of patches *n* and *m* and to compute the Green's function φ^{GR} directly with the required precision. At this moment I have the approximation function of mentioned dependency tested only for patches that partial derivatives $\delta h(x,y)/\delta x$ and $\delta h(x,y)/\delta y$ are *0*. On the figure you can see the plot function of number of sub-patches *n* for interval of relative tolerances c_{est}. from 0.5 to 10^{-5}. The relative tolerance is defined for complex φ^{GR} by:

$$c_{est} = \frac{|C_{est}|}{|\varphi_i^{GR}|} \tag{16}$$

Approximation function is because of the shape of dependency the logarithmic progression:

$$\overline{n} = c_0 + \sum_{i=1}^{I} c_i \cdot \ln c_{est}{}^{p+i} \tag{17}$$

where vector ***c*** is vector of coefficients having the length *I+1*. *I* is the number of progression's members. It seems to be reasonable to choose *I=3* and arbitrary parameter *p=3*. Then approximation function has the form:

$$\overline{n} = c_0 + c_1 \ln c_{est}{}^4 + c_2 \ln c_{est}{}^5 + c_3 \ln c_{est}{}^6 \tag{18}$$

The coefficient-vector *c* was found by least square method:

$$c = \begin{bmatrix} 3.6936 \\ 4.9558 \cdot 10^{-2} \\ 8.0617 \cdot 10^{-3} \\ 5.1790 \cdot 10^{-4} \end{bmatrix} \tag{19}$$

The vector of coefficients was computed as an arithmetic average of vectors evaluated for the patch of dimensions $\Delta x_{sub}=1mm$ and $\Delta y_{sub}=1mm$ "illuminated" from several directions and ranges (changing vector ***R***) and by number of frequencies from X and Ku bands.

CONCLUSION

Described algoithm was practically tested in Matlab as a set of m-functions. The most significant shortcoming of it was the computation speed. For instance I have launched this computation on rough surface of dimensions 30 cm by 30 cm divided on to square 1 mm – patches. When requested error estimation has been between 10^{-3} and 10^{-4}, the number of elements in every patch would be 100 by 100. You can imagine the consumption of computation power needed to solve the numerical integration using this algorithm.

On the other hand the advantage of used approach is the flexibility of computation. There exists the possibility to extend algorithm and to compute curve-shaped surface of the patches. That is the way to compute 3D-cubic spline approximations of curved surfaces, of course with the redefined approximation function.

REFERENCES

[1] Oglivy,J. A., Theory of wave scattering from random rough surfaces. Institute of physic publishing, Bristol and Philadelphia, 1991.

[2] MATLAB Electronic Documentation System, MATLAB Help.

[3] Ruck, G. T., Barrick, D. E., Stuart, W. D., Krichbaum, C. K., Radar Cross Section Handbook, Plenum Press, New York, 1970.