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# DESIGN OF THE CHEBYSHEV BR FILTER FOR THE ELASTIC AIRCRAFT LONGITUDINAL STABILITY AUGMENTATION SYSTEM

## ABSTRACT

In classical automatic flight control system's theory aircraft is considered as the rigid-body one. The controller automatically stabilizing the aircraft spatial motion is designed for the nominal plant. In real flight aircraft behaves elastically. Any external force or moment results in the aircraft elastic motion. The most common mathematical representation of the fuselage bending motion is the transfer function method. If to consider the aircraft elevator angular deflection for the input and the pitch rate as the output the model of the elastic motion can be considered as additive uncertainty. The purpose of the authors is to design a filter for hypothetical aircraft pitch rate gyroscope. For the solution of this problem a new MATLAB® M-file was created by the authors.

## INTRODUCTION

Due to their main features space and air vehicles are elastic ones. Airplanes are maneuvering in the three dimensional space and they must be considered as elastic vehicles. Aeroelasticity is in the focus of attention since many decades. Knowledge of the aircraft elastic motion is important for designers from the point of view of the sensor location upon the aircraft. If elastic motion results in the error of rate sensing it is necessary to filter electric signals of the sensors. Many aircraft flight control system is equipped with filters designed for filtering unwanted signals from the first and second elastic overtone [5]. One of the possible methods is the classical representation based upon the transfer function method. The most modern method for the elastic motion modeling is the state space representation, which allows to consider the aircraft as the multi input - multi output (MIMO) system. In this paper transfer function method is used for representation of the high frequency dynamics of the elastic aircraft.

## MATHEMATICAL MODEL OF THE ELASTIC AIRCRAFT

During the mathematical modeling of the elastic aircraft the fuselage and the wings elastic motion can be analyzed. The aircraft fuselage is considered as a simple rod. The fuselage bending motion equation is given in [2, 3, 6] to be:

$$\frac{d^2 q_i(t)}{dt^2} + 2\xi_i \omega_i \frac{dq_i}{dt} + \omega_i^2 q_i(t) = K_1 \delta_E(t) \tag{1}$$

where $K_1$ is constant gain, $\omega_i$ is the natural frequency of the undamped oscillation of the i*th* elastic mode, $\xi_i$ is the damping ratio of the undamped oscillation of the i*th* elastic mode, $q_i$ is the generalized coordinate of the i*th* elastic mode. Taking Laplace transform of eq. (1) respecting zero initial conditions we have:

$$\left(s^2 + 2\xi_i \omega_i s + \omega_i^2\right) q_i(s) = K_1 \delta_E(s) \tag{2}$$

It is easily can be seen that pitch rate generated by fuselage elastic motion can be determined as follows:

$$\omega_{z_E}(s) = \sum_{i=1}^{\infty} \frac{sK_i}{s^2 + 2\xi_i \omega_i s + \omega_i^2} \delta_E(s) \tag{3}$$

where $K_i$ is the gain of the i*th* elastic degree of freedom. In [4, 5, 6] parameters of the 1st and the 2nd overtones of the hypothetical fighter fuselage bending motion are given as follows:

$$K_1 = 10\ s^{-2},\ \omega_1 = 10\ s^{-1},\ \xi_1 = 0{,}05;\ K_2 = 5\ s^{-2},\ \omega_2 = 20\ s^{-1},\ \xi_2 = 0{,}02 \tag{4}$$

Later it will be supposed that the longitudinal motion control system is affecting only the short period motion. The simplified mathematical model of the longitudinal motion of the aircraft is given by the following equation [1, 6, 7]:

$$\omega_{Z_R}(s) = -\frac{A(1 + sT_\theta)\omega_\alpha^2}{s^2 + 2s\xi_\alpha \omega_\alpha + \omega_\alpha^2} \delta_E(s) \tag{5}$$

In eq. (5) for the flight conditions H=1000 m and M=0.4 let us consider the following parameters of the aircraft [5, 6]:

$$A = 1{,}5\,s^{-1}\,;\,T_\theta = 2\,s\,;\,\omega_\alpha = 5\,s^{-1}\,;\,\xi_\alpha = 0{,}5 \qquad (6)$$

The output signal of the pitch rate gyro can be determined as sum of the rigid and elastic aircraft output signals defined by eqs (3) and (5):

$$\omega_Z(s) = \omega_{Z_E}(s) + \omega_{Z_R}(s) \qquad (7)$$

The aeroelastic aircraft model built by eqs. (3), (5) and (7) is represented in Figure 1.

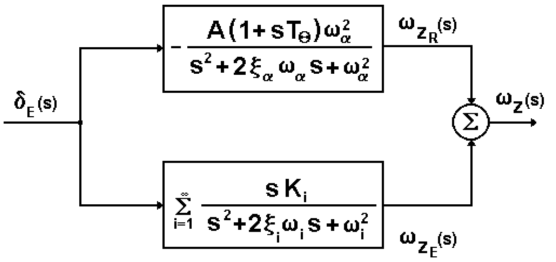


Fig. 1. The Aircraft Rigid Model and Elastic Model

Sign '-' in rigid aircraft transfer function is for direction measuring between elevator deflection and the pitch rate. Elevator deflection is supposed to be positive if leads to negative pitch rate. If to neglect this sign in pitch rate damper the feedback must be positive.

## TIME DOMAIN ANALYSIS OF THE UNCONTROLLED AIRCRAFT

Let us consider the aircraft model defined by eq (5) and (6). Eigenvalues and dynamic performances of the aircraft are as follows:

$$\lambda_{1,2} = -2{,}5 \pm 4{,}33i, \xi = 0{,}5, \omega = 5 rad/s \qquad (8)$$

The uncontrolled rigid and the uncontrolled elastic aircraft was analized in the time domain. Result of the computer simulation can be seen in Figure 2. From Figure 2 it can be seen that the uncontrolled aircraft transient response has large overshoot and response time. If the plant model is perturbed with elastic motion ovestones given by eq. (4) the step response of the uncontrolled aircraft is highly oscillatory one. Pitch rate oscillates around step response of the rigid aircraft.

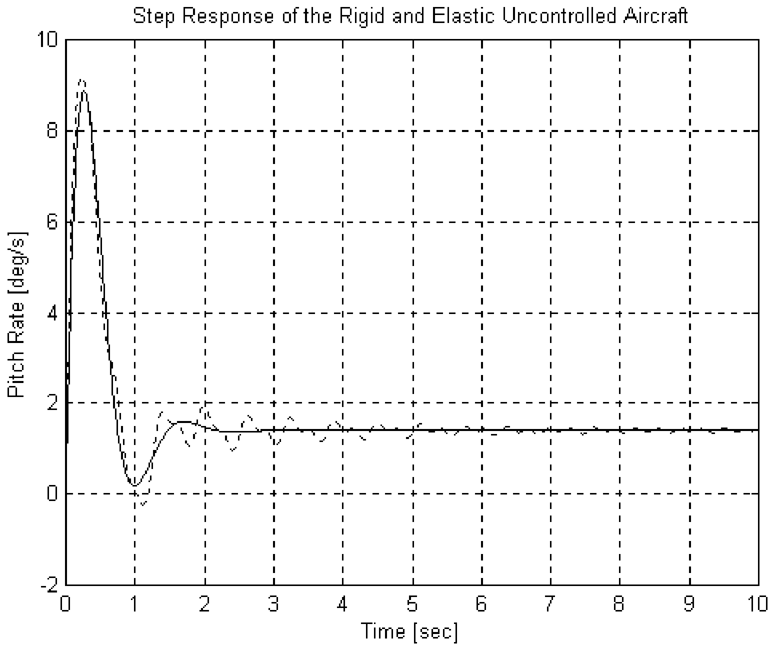


Fig. 2. Pitch Rate Step Responses
solid: uncontrolled rigid aircraft dash: uncontrolled elastic aircraft

# FREQUENCY DOMAIN ANALYSIS OF THE UNCONTROLLED AIRCRAFT

Bode diagram of the additive uncertainty represented by the high frequency dynamics of the aircraft elastic motion can be seen in Figure 3. Uncertainty gain has resonance peak at 10 $s^{-1}$ and at 20 $s^{-1}$. These peaks are developed by the D-lag in the numerator of eq. (3). Both in low and high frequency domain uncertainty gain is small.

The additive uncertainty affects the frequency domain behavior of the open loop stability augmentation system. Results of the computer simulation can be seen in Figure 4. During computer simulation unity gains for the controller and the pitch rate gyro were supposed.

From Figure 4 can be seen the effect from elastic motion dynamics, which can be considered for additive uncertainty. At the resonance frequencies of 10 $s^{-1}$ and 20 $s^{-1}$ the gain and the phase angle have peaks in their values. The open loop gain and the phase angle are increased only at the resonance frequency and in its nearest domain.

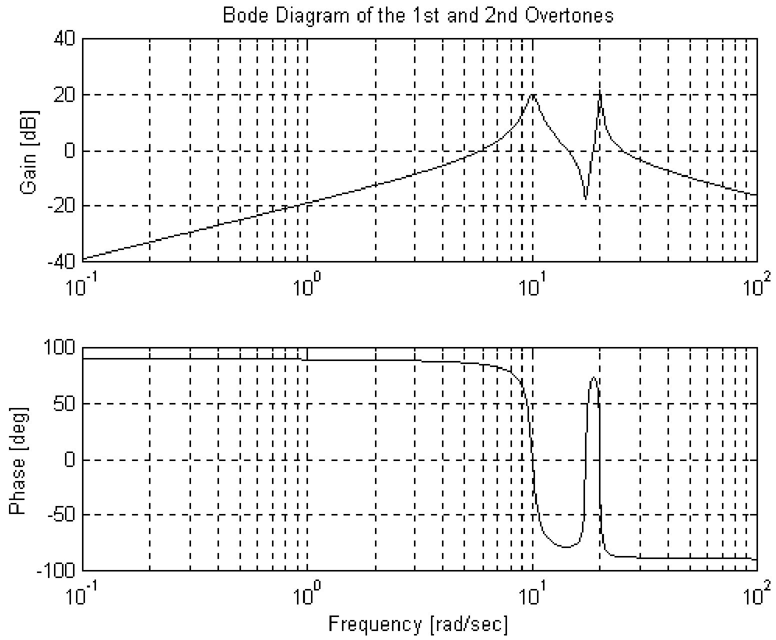


Fig. 3. Elastic Overtones Modeled as Additive Uncertainty

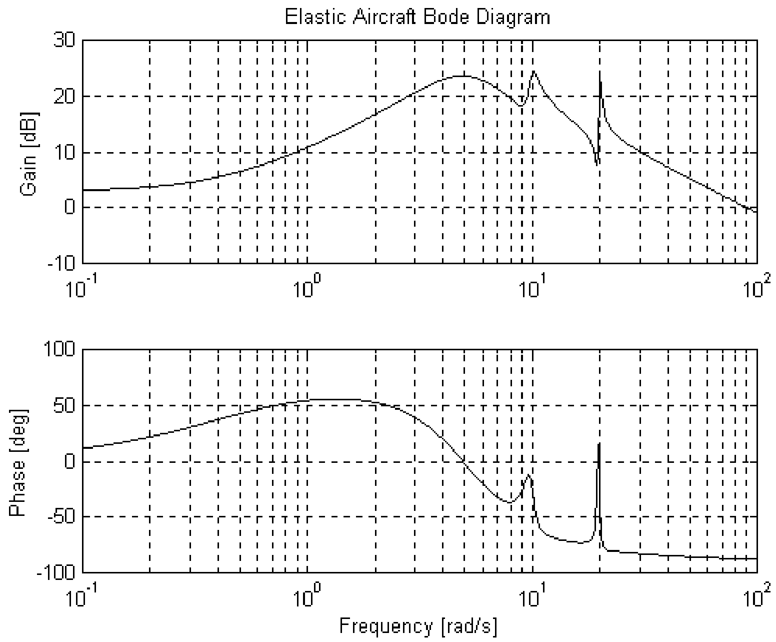


Fig. 4. Bode Diagram of the Open Loop Perturbed System

# BR FILTER DESIGN FOR PITCH RATE SENSOR

From Chapter "Mathematical Model of the Elastic Aircraft" it is evident that dynamics of the elastic motion of the aircraft fuselage is defined with the proportional-differential second order term (see eq. 3). For filtering of the output signals from pitch rate sensors there are some types of filter transfer functions given in [6] as follows:

$$Y_{F_1}(s) = \frac{(1+sT_1)(1+sT_2)}{(1+sT_3)(1+sT_4)} \tag{9}$$

$$Y_{F_2}(s) = \frac{s^2 + 2\xi_i\omega_i s + \omega_i^2}{s^2 + 2\xi_T\omega_i s + \omega_i^2} \tag{10}$$

Band rejection transfer function $Y_{F_1}$ is for decreasing gain overshoots generated by elastic overtones. Filter design means finding appropriate time constants $T_1, T_2, T_3$ and $T_4$ for determining band rejection transfer function. The other possible transfer function is given by eq. (10). In this equation $\xi_T$ is used as tuning parameter. From eq. (10) it is easily can be determined that for $\xi_i \ll \xi_T$ takes place the following equation:

$$\left|Y_{F_2}(j\omega)\right|_{\omega=\omega_i} \ll 1; \; \left|Y_{F_2}(j\omega)\right|_{\omega\neq\omega_i} \approx 1 \tag{11}$$

Tuning parameter $\xi_T$ must be found heuristically for determination of the filter transfer function. At the first stage filters were derived for resonance frequencies of 10 $s^{-1}$ and 20 $s^{-1}$. Filters preliminary designed was tested and it was derived that transfer function given by eq. (10) provides not enough bandwidth for rejecting elastic motion gain overshoots. For increase of bandwidth of the band rejection filters for each overtone resonance frequency there was applied series connection of two filters adjusted for 8,3 $s^{-1}$ and 10,6 $s^{-1}$. Transfer functions derived for these frequencies are:

$$Y_{11}(s)_{\omega=8,3} = \frac{s^2 + 2{,}65s + 68{,}89}{s^2 + 2{,}1s + 112{,}36}, \; Y_{12}(s)_{\omega=10,6} = \frac{s^2 + s + 112{,}36}{s^2 + 2{,}2s + 112{,}36} \tag{12}$$

The second overtone is rejected by series connection of filters adjusted for 19,6 $s^{-1}$ and 20,1 $s^{-1}$. These transfer functions are:

$$Y_{21}(s)_{\omega=19,6} = \frac{s^2 + 2,6s + 384,16}{s^2 + 0,95s + 384,16}, \quad Y_{22}(s)_{\omega=20,1} = \frac{s^2 + 2,6s + 384,16}{s^2 + 0,95s + 384,16} \qquad (13)$$

Using eqs. (12) and (13) band rejection filters were tested in frequency domain. Results of the computer simulation can be seen in Figure 5.

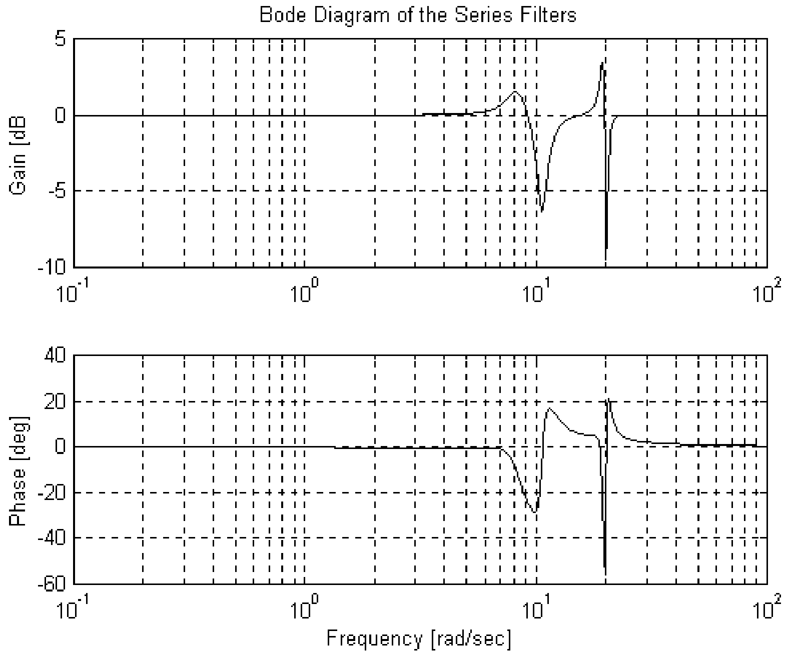


Fig. 5. Frequency Domain Behavior of the BR Filters

## TIME DOMAIN ANALYSIS OF THE AIRCRAFT LONGITUDINAL STABILITY AUGMENTATION SYSTEM

Dynamic performances of the controlled aircraft e.g. the damping ratio must be between 0,6 and 0,8 [7]. For providing desirable dynamic performances the pole placement method can be used. Pole placement is realized with state feedback by the pitch rate. The pitch rate damper is built using sensor, controller and hydraulic actuator. In conventional stability augmentation systems the pitch rate sensor is the electro-mechanical device. Sensor dynamics can be represented as the proportional second order lag. Assuming high natural frequency of the rate gyro it can be modeled as a simple proportional lag with unity gain $K_s$. The compensator is supposed to be proportional lag $K_c$. During analysis of the pitch rate it is supposed that hydraulic actuator has fast response to input signals without any time delay. The block diagram of the longitudinal stability augmentation system when the first and the second overtone of the aircraft elastic motion are taken into account can be seen in Figure 6.

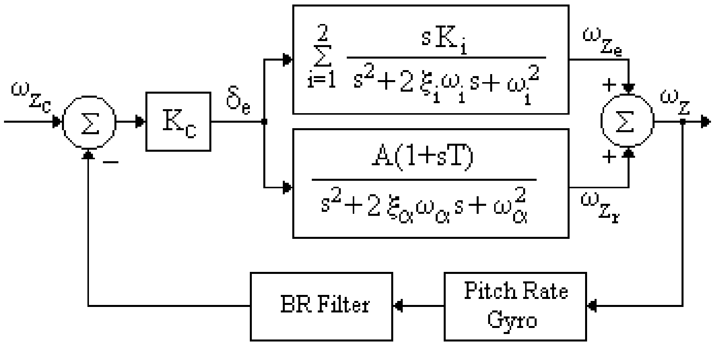


Fig. 6. Longitudinal Motion Stability Augmentation System

The longitudinal stability augmentation with the unity gain controller was analyzed in the time domain. Results of the computer simulation can be seen in Figure 7. From this figure it is easily can be stated that the rigid and the elastic aircraft behavior are very close to each other. For having appropriate time domain dynamic performances closed loop system must be adjusted varying controller gain $K_c$.

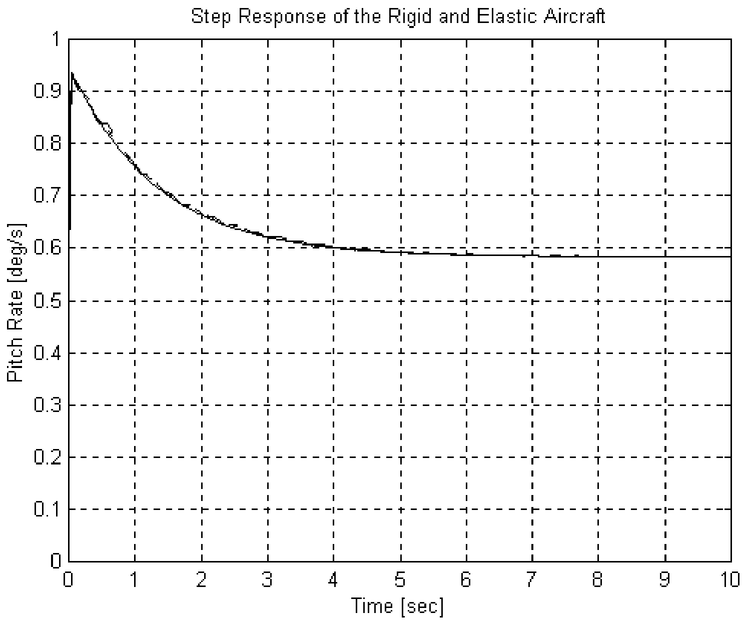


Fig. 7. Transient Behavior of the Aircraft

'—': rigid aircraft; '- -': elastic aircraft; '…': elastic aircraft with filter

# FREQUENCY DOMAIN ANALYSIS OF THE AIRCRAFT LONGITUDINAL STABILITY AUGMENTATION SYSTEM

In Chapter “Frequency Domain Analysis of the Uncontrolled Aircraft” Bode diagram of the elastic aircraft was shown. Applying filter given by eqs. (12) and (13) the open loop control system was analyzed in the frequency domain. Results of the computer simulation can be seen in Figure 8.

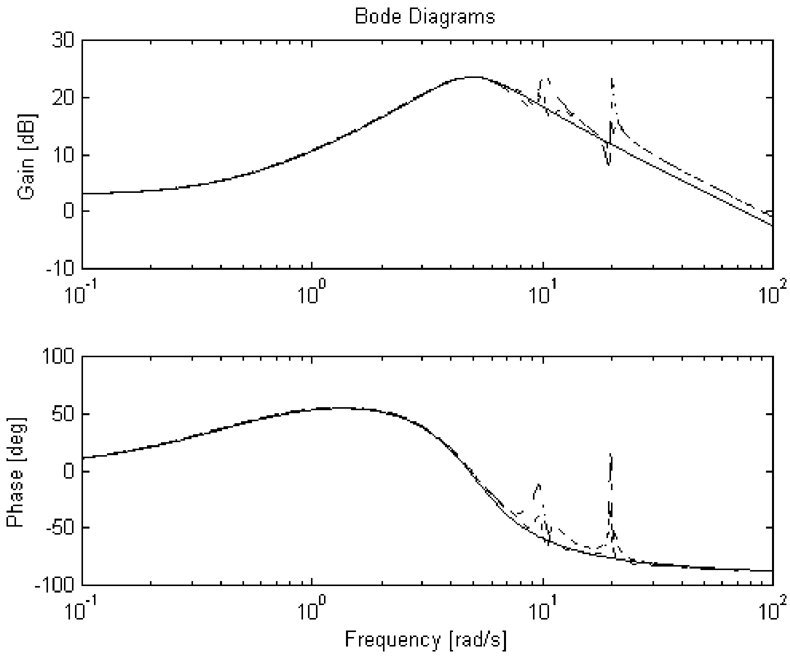


Fig. 8. Open Loop System Bode Diagrams
‘—‘: rigid aircraft; ‘- -‘: elastic aircraft; ‘…’: elastic aircraft with filter

From Figure 8 it is easily can be seen that filters adjusted for resonance frequencies of the first and second overtones of the aircraft decrease overshoots of the open loop gain. In frequency range beyond that of the second overtone there is some increase of the open loop gain. Overshoots in phase angle also decreased and there are deviations in it only at resonance frequencies.

## CONCLUSIONS

The paper deals with problems of mathematical modeling of aeroelastic aircraft and with problems of signal filtering of rate gyro output signal. The BR filters for elastic aircraft were designed for the hypothetical aircraft longitudinal stability augmentation system. Filters provide for the closed loop and for the open loop control system ‘good’ dynamic performances. For solution of the problem a new MATLAB® M-file was created by the authors.


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