# Examination of Complex Traffic Dynamic Systems

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Abstract: A validated, quick and cost-effective alternative method can be applied to analyze driver load, which is based on the track record of the city, traffic simulation model and parameters of the vehicle and driver characteristics.

*Keywords: positive systems, complex traffic dynamic systems, macroscopic model, traffic simulation, alternative method, examination of the network processes and the vehicles in the network,*

## Introduction

The road traffic is an extremely complex dynamic system. The up to date examination of this system requires complex methods. Nowadays the examination of the network processes and the vehicles in the network in the form of a system is inseparable.

For this purpose a test methodology is proposed based on the theory of positive systems, where, in essence, the model is a macroscopic model.

The terms of controllability and observability of positive systems are not derivable from the known methods applied in general systems [1]. The problem is particularly true when a non-negative co-domain is required not only for the states, but for the control input sign too. Therefore, describing the road system processes as pure positive systems is not a trivial task from the control engineering point of view. The control task in this case means that the system must be controlled from a state to another so, that the states remain non-negative values during the transitions too [2].

This environment - despite the fact that it is macroscopic model - will be suited to describe a real traffic process from any point A to a selected point B of the network at any departure time taking real transport processes (lights, congestion, etc.) into account, [3]. In addition to the route recommendations this is an important result in the area of intelligent vehicle studies (e.g., analysis of vehicle dynamics, engineering, environmental loads, emissions analysis, etc.) as it is possible to perform the calculations for a large number of vehicles very quickly at different times and places.

The trajectories measured in real traffic are suitable for validation of the network model as well.

## The model

In the referred contributions a constructed network is discussed, which is bounded by a closed curve [4-8]. It contains n sectors of internal network and m sectors from external network. We assume, the external sectors have direct connection to internal sectors and their state is known by measurements. The differential equation system is the following:

$$\dot{x} = \langle L \rangle^{-1} [K_{11}(x,s)x + K_{12}(x,s)s] \tag{1}$$

where $x \in \Re^n$, $\forall$ $x_i \in [0,1]$, (i=1,2,…,n), $\dot{x} \in \Re^n$, $s \in \Re^m$, $\forall$ $s_i \in [0,1]$, (i=1,2,…,m), $L$ = diag{$l_1$, ... ,$l_n$}, $l_i$ length of road sections in the main diagonal ($\forall l_i > 0$, i=1,2,…,n), $K_{11} \in \Re^{nxn}$, $K_{12} \in \Re^{nxm}$.

The operation of the network is determined by $K_{11}$ and $K_{12}$ relational matrixes. These matrixes assign the existence of the relationship between every sector of the system, and at the same time it represents the differential equation system describing the dynamic operation of the sections, so the constructed network.

## Analysis of the velocity processes

It is assumed that to $\forall x_i$ , ($x_i \in [0,1]$, n=1,2,…n) state parameter $v_i \geq 0$ velocity value can also be assigned by the application of an $f_i$ function continuously differentiated by $x_i$:

$$v_i = f_i(x_i(t)) \tag{2}$$

By the retrieval of the unique velocity processes and application of a driver-vehicle model from the macroscopic network model the engine power demand and the exhaust gas emission can be examined.

The model can be validated by velocity processes. The model was validated in Budapest on the boulevard between Petőfi Bridge and Nyugati Square [9-11].

In different simulation times the examined route was reviewed by vehicles with GPS and during the measurements the real velocity profiles were also recorded. The comparison of velocity-time diagram acquisition by simulation and vehicle measurements obviously showed that time diagrams should be considered as realizations of a stochastic process and investigated by statistical analysis.

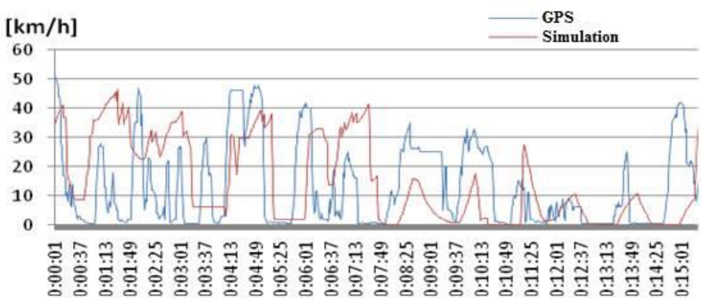


*Figure 1:* Simulation based velocity profiles measured by GPS

By large nonparametric statistical analysis, so called, homogeneity analysis was proven that the two samples got by simulation and measurement are considered homogeneous at 95%. From the velocity data the same result has come concerning vehicle engine power demand.

During the model validation it could be stated that the model makes vehicle process acquisition possible, which reflects reality [12].

Based on the above the vehicle profiles can be directly acquired by the arbitrary trajectories of the network as well. In the following vehicle dynamics is analyzed along an assigned trajectory from an optional A to B in $t_0$. Along the trajectory the X(t) path-time function and T (end time) can be calculated. The two-variant velocity function V(t, X) belonging to a rectified X trajectory and t time point can be calculated by the state equation (see Figure 2.)

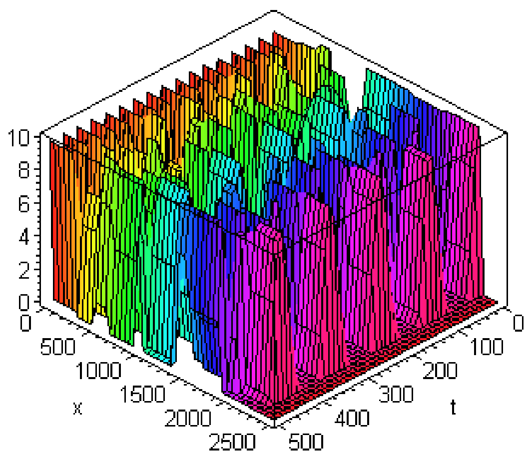


*Figure 2: Two-variant velocity function V(t, X) belonging to a rectified X trajectory and t time point*

By the solution of the following integral equation and the two-variant velocity function V(t, X) the X(t) path-time function can be determined:

$$x(t) = \int_{t_0}^{t} V(\tau, x(\tau))\, d\tau \tag{3}$$

The problem needs the solution of the following first order nonlinear differential equation at the $X(t_0)=x_0$ initial condition:

$$\frac{dX(t)}{dt} = V(t, X(t)) - V(t_0, X(t_0)) \tag{4}$$

$$x(t_0) = x_0$$

The solution is given by the application of numerical method, e.g. Figure 3.

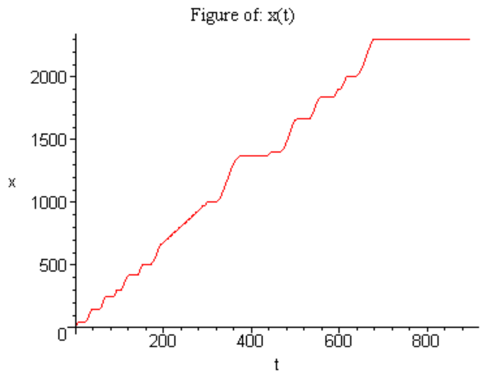


*Figure 3: Path-time diagram*

After $t_1$ end time there's no increase in X(t), e.g. end time $T=t_1-t_0$;

Note: If in case of more trajectories the optimal end point reach is investigated, the problem needs a solution of a variation calculation task. Along every trajectory the path X run until the t time point results in an X(t) path function, to which in point B belongs a T time and this mapping provides the real functional:

$$J:\ X(t) \rightarrow T \tag{5}$$

Therefore large traffic network model can be applied to real time route planning taking also the traffic changes into consideration.

## Analysis of acceleration processes

On the basis of known velocity processes the longitudinal accelerations can also be calculated in the optional i. section of the traffic model:

$$\dot{v}_i(t) = a(t) = \frac{df_i(x_i(t))}{dx_i} \cdot \dot{x}_i(t) = f_i' \cdot \dot{x}_i \tag{6}$$

(i=1,2, … ,n). The velocity vector in the whole internal domain:

$$v(t) = f(x(t)) = \begin{bmatrix} f_1(x_1) \\ f_2(x_{2)} \\ .... \\ f_n(x_n) \end{bmatrix} \tag{7}$$

After derivation the acceleration vector can be formed as well:

$$a(t)=\dot{v}(t)=\begin{bmatrix} f_1'(\dot{x}_1) \\ f_2'(\dot{x}_2) \\ \dots \\ f_n'(\dot{x}_n) \end{bmatrix}=\begin{bmatrix} f_1' & & & \\ & f_2' & & \\ & & \dots & \\ & & & f_n' \end{bmatrix}\cdot\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dots \\ \dot{x}_n \end{bmatrix} \quad (8)$$

Based on the system state equation the continuous acceleration vector can be directly calculated:

$$a(t)=\left\langle f_i' \right\rangle \cdot \dot{x}=\left\langle \frac{f_i'}{l_i} \right\rangle \cdot [K_{11}(x,s)x+K_{12}(x,s)s] \quad (9)$$

Where: $a\in\Re^{n}$, $\left\langle f_i' \right\rangle = diag\{f_1', f_2',..., f_n'\}$.

## Conclusions

Along the assigned trajectory the X(t) path-time function can be calculated, which gives the location of the vehicle in t time point. Also the velocity and acceleration are known in t time point in the actual section of the trajectory.

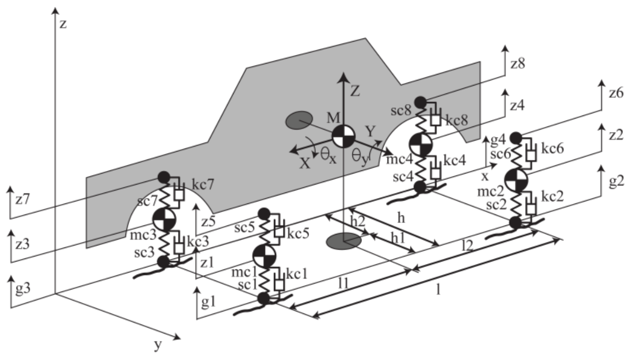


*Figure 4: Vehicle model*

According to the above the traffic network model already gives crucial data to vehicle dynamics analyses. Because of its fast use and applicability at one time to large number of vehicles, this analysis has a big effect on further automotive researches [13-17].

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