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On the Partial Approximation of Sets

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Abstract 1 According to Pawlak's classic rough set theory the vagueness of a subset of a finite universe U is defined by the difference of its upper and lower approximations with respect to an equivalence relation on U. A natural way of the generalization of this idea is that the equivalence relation is replaced by either any other type of binary relations on U or an arbitrary covering of the universe. In this paper, our starting point will be an *arbitrary* family of subsets of an *arbitrary* universe U. Within this framework, we shall investigate a possible generalization of Pawlak's idea. Both Pawlak's rough set theory and our approach can extensively be applied in medical informatics.

Keywords: vagueness, rough set theory, partial approximation of sets, medical in-formatics

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1. Introduction

The rough set theory was introduced by the Polish mathematician Z. Pawlak in the early 1980s [15, 16]. Let U be a *finite* set of distinguishable objects which is called the universe of discourse, and $\varepsilon \subseteq U \times U$ be an equivalence relation on U. The elements of partition generated by ε are called ε-elementary sets. An ε-elementary set can be viewed as a set of indiscernible objects characterized by the same available information about them [18, 20]. Any union of ε-elementary sets is referred to as *definable* set. An arbitrary subset $X \subseteq U$ may not necessarily be a union of some ε-elementary sets. However, it can be naturally approximated by two sets, by the union of ε-elementary sets that are subsets of X, called the lower ε-approximation of X, and by the union of ε-elementary sets that have a nonempty intersection with X, called the upper ε-approximation of X.

The basic idea of Pawlak's rough set theory is that the vagueness [12, 17, 19, 20] of a set is described by the difference of its upper and lower ε-approximations called the ε-boundary of the set. A set is *rough* if its ε-boundary is nonempty.

Using partitions, however, is a strict requirement. Moreover, in practice, there are attributes which do not characterize all members of an observed collection of objects [6, 13].

A natural way of the generalization of Pawlak's idea is that the equivalence relation is replaced by any other type of binary relations on U [10, 11, 24]. Another generalization is the assumption that the starting point is an arbitrary covering of the universe [1, 23, 26, 27]. In this paper, our starting point will be an *arbitrary* family of subsets of an *arbitrary* universe U. We will not assume whether this family of sets covers the universe or the universe is finite.

The paper is organized as follows. In Section 2 we summarize the basic notations used throughout the paper. Section 3 presents the basic concepts and their properties of the classical Pawlak's rough set theory. Only those facts which are important from the point of view of the generalization will be considered. The major contributions of this paper are covered in Section 4 which summarizes the basic principles of the partial approximation of sets.

2. Basic Notations

Let U be any set. The powerset of U is 2^U. If $\mathfrak{A} \subseteq 2^U$, then the union of $\mathfrak{A}$ is $\bigcup \mathfrak{A} = \{x \mid \exists A \in \mathfrak{A}(x \in A)\}$, the intersection of $\mathfrak{A}$ is $\bigcap \mathfrak{A} = \{x \mid \forall A \in \mathfrak{A}(x \in A)\}$. If $\mathfrak{A}$ is an empty family of sets we define $\bigcup \emptyset = \emptyset$ and $\bigcap \emptyset = U$.

If ϵ is an arbitrary binary relation on U, let $[x]_\epsilon$ denote the ϵ-related elements to x, i.e., $[x]_\epsilon = \{y \in U \mid (x, y) \in \epsilon\}$. They are called ϵ-elementary sets, and the family of $[x]_\epsilon$ is denoted by U/ϵ.

Let X and Y be nonempty sets and $f : X \to Y$ be a map. If $\mathtt{dom} f = X$ then f is *total*, if $\mathtt{dom} f \subsetneqq X$ then f is *partial*. If f is a partial map, then $\mathtt{dom} f = \emptyset$ is allowed. For the purpose of simplicity we will talk about partial maps without direct references to their partial properties. However, statements with respect to partial maps always concern their restrictions to their domains.

A nonempty set P together with a partial order $\leqslant$ on P is called a *poset*, in symbols $(P, \leqslant)$. Let $(P, \leqslant_P)$ and $(Q, \leqslant_Q)$ be two posets. A map $f : P \to Q$ is *monotone* if $x \leqslant_P y \Rightarrow f(x) \leqslant_Q f(y)$, *order-embedding* if $x \leqslant_P y \Leftrightarrow f(x) \leqslant_Q f(y)$, and *order-isomorphism* if f is an order-embedding onto Q. In general, monotone maps are many-to-one correspondences. An order embedding is always monotone and injective. Hence, f is an order-isomorphism if and only if f is a bijection, and both f and f^{-1} are monotone.

3. Basics of Pawlak's Rough Set Theory

Definition 3.1. The pair (U, ε) where U is a finite universe of discourse and ε is an equivalence relation on U is called *Pawlak's approximation space.*

Equivalence classes generated by the equivalence relation ε are ε*-elementary sets.* A subset $X \subseteq U$ is ε*-definable*, if it is a union of ε-elementary sets, otherwise X is ε*-undefinable.* By definition, the empty set is considered to be an ε-definable set.

Let $\mathfrak{D}_{U/\varepsilon}$ denote the family of ε-definable subsets of U.

Remark 3.2. The idea of approximation *space* is a bit younger than Pawlak's initial works. For an evolutionary survey of approximation spaces, see [20].

The following statement is elementary, however, in the context of Pawlak's rough set theory it is an important fact. For the sake of simple reference, it is formulated in a lemma. It follows just from the fact that the partition U/ε generated by ε consists of nonempty pairwise disjoint subsets of U.

Lemma 3.3. $\forall \mathfrak{X} \in 2^{U/\varepsilon}\ \forall X \in U/\varepsilon\ (X \subseteq \bigcup \mathfrak{X} \Leftrightarrow X \in \mathfrak{X})$*.*

Clearly, by Definition 3.1, $\mathfrak{D}_{U/\varepsilon}$ contains the empty set and is closed under complementation and unions. According to Lemma 3.3, it is also closed under intersections, i.e., $\mathfrak{D}_{U/\varepsilon}$ is a σ-algebra with basis U/ε.

Proposition 3.4 ([3], Theorem 8) Let (U, ε) be a Pawlak's approximation space.

Then the map $\mathfrak{u}_\varepsilon : 2^{U/\varepsilon} \to \mathfrak{D}_{U/\varepsilon}, \mathfrak{X} \mapsto \bigcup \mathfrak{X}$ is an order isomorphism between $(2^{U/\varepsilon}, \subseteq)$ and $(\mathfrak{D}_{U/\varepsilon}, \subseteq)$.

Corollary 3.5. *Any* ε*-definable subset* $D \in \mathfrak{D}_{U/\varepsilon}$ *of* U *can be written uniquely in the following form:*

$$D = \bigcup \mathfrak{X}, \text{ where } \mathfrak{X} = \{X \mid X \in U/\varepsilon, X \subseteq D\} \in 2^{U/\varepsilon},$$

that is, there is no other $\mathfrak{X}' \in 2^{U/\varepsilon}$ *satisfying* $D = \bigcup \mathfrak{X}'$*.*

Proof. Since $D \in \mathfrak{D}_{U/\varepsilon}$, thus $D = \bigcup \mathfrak{X}$ immediately holds by Lemma 3.3. However, $\mathfrak{u}_\varepsilon$ is a bijection, so $\mathfrak{u}_\varepsilon^{-1}(D) \in 2^{U/\varepsilon}$ is unique and $\mathfrak{u}_\varepsilon^{-1}(D) = \mathfrak{X}$. □

In Pawlak's approximation spaces, lower and upper approximations of $X \in 2^U$ can be defined in three equivalent forms [21, 22, 25].

Definition 3.6. Let (U, ε) be a Pawlak's approximation space and $X \in 2^U$ be any subset of U.

The *lower* ε*-approximation* of X is

$$\begin{aligned} \underline{\varepsilon}(X) &= \{x \in U \mid [x]_\varepsilon \subseteq X\}, & (3.1a)\\ &= \bigcup\{Y \mid Y \in U/\varepsilon, Y \subseteq X\} & (3.1b)\\ &= \bigcup\{Y \mid Y \in \mathfrak{D}_{U/\varepsilon}, Y \subseteq X\}, & (3.1c) \end{aligned}$$

the *upper* ε*-approximation* of X is

$$\begin{aligned} \overline{\varepsilon}(X) &= \{x \in U \mid [x]_\varepsilon \cap X \neq \emptyset\}, & (3.2a)\\ &= \bigcup\{Y \mid Y \in U/\varepsilon, Y \cap X \neq \emptyset\}, & (3.2b)\\ &= \bigcap\{Y \mid Y \in \mathfrak{D}_{U/\varepsilon}, X \subseteq Y\}. & (3.2c) \end{aligned}$$

It follows just from the definitions that $\underline{\varepsilon}(X), \overline{\varepsilon}(X) \in \mathfrak{D}_{U/\varepsilon}$, in addition the maps $\underline{\varepsilon}, \overline{\varepsilon} : 2^U \to \mathfrak{D}_{U/\varepsilon}$ are total and many-to-one.

Proposition 3.7 Let (U, ε) be a Pawlak's approximation space and $X \in 2^U$ be a subset of U. The sets $\underline{\varepsilon}(X)$, $\overline{\varepsilon}(X)$ can be written uniquely in the following forms:

$$\begin{aligned} \underline{\varepsilon}(X) &= \bigcup \underline{\mathfrak{X}}, \text{ where } \underline{\mathfrak{X}} = \{Y \mid Y \in U/\varepsilon, Y \subseteq X\} \in 2^{U/\varepsilon},\\ \overline{\varepsilon}(X) &= \bigcup \overline{\mathfrak{X}}, \text{ where } \overline{\mathfrak{X}} = \{Y \mid Y \in U/\varepsilon, Y \cap X \neq \emptyset\} \in 2^{U/\varepsilon}, \end{aligned}$$

that is, there are no other $\underline{\mathfrak{X}}', \overline{\mathfrak{X}}' \in 2^{U/\varepsilon}$ satisfying $\underline{\varepsilon}(X) = \bigcup \underline{\mathfrak{X}}'$ and $\overline{\varepsilon}(X) = \bigcup \overline{\mathfrak{X}}'$.

Proof. According to Definition 3.6 (3.1b), (3.2b), we only have to prove the uniqueness.

$\underline{\varepsilon}(X)$, $\overline{\varepsilon}(X) \in \mathfrak{D}_{U/\varepsilon}$, and so, by Proposition 3.4, $\mathfrak{u}_\varepsilon^{-1}(\underline{\varepsilon}(X))$ and $\mathfrak{u}_\varepsilon^{-1}(\overline{\varepsilon}(X))$ are unique. Hence, by Lemma 3.3, we get

$$\begin{aligned} \mathfrak{u}_\varepsilon^{-1}(\underline{\varepsilon}(X)) &= \{Y \mid Y \in U/\varepsilon, Y \subseteq \underline{\varepsilon}(X)\}\\ &= \{Y \mid Y \in U/\varepsilon, Y \subseteq \bigcup\{Y' \mid Y' \in U/\varepsilon, Y' \subseteq X\}\}\\ &= \{Y \mid Y \in U/\varepsilon, Y \in \{Y' \mid Y' \in U/\varepsilon, Y' \subseteq X\}\}\\ &= \{Y \mid Y \in U/\varepsilon, Y \subseteq X\} = \underline{\mathfrak{X}}.\\ \mathfrak{u}_\varepsilon^{-1}(\overline{\varepsilon}(X)) &= \{Y \mid Y \in U/\varepsilon, Y \subseteq \overline{\varepsilon}(X)\}\\ &= \{Y \mid Y \in U/\varepsilon, Y \subseteq \bigcup\{Y' \mid Y' \in U/\varepsilon, Y' \cap X \neq \emptyset\}\}\\ &= \{Y \mid Y \in U/\varepsilon, Y \in \bigcup\{Y' \mid Y' \in U/\varepsilon, Y' \cap X \neq \emptyset\}\}\\ &= \{Y \mid Y \in U/\varepsilon, Y \cap X \neq \emptyset\} = \overline{\mathfrak{X}}. \end{aligned}$$

□

Basic properties of lower and upper ε-approximations can be found, e.g, in [10, 16]. Here we cite only a few among them which will be important in the following.

Proposition 3.8 ([16], Proposition 2.1, point a)) Let (U, ε) be a Pawlak's approximation space. Then $X \in \mathfrak{D}_{U/\varepsilon}$ if and only if $\underline{\varepsilon}(X) = \overline{\varepsilon}(X)$.

Proposition 3.9 ([16], Proposition 2.2, points 1, 9, 10) Let (U, ε) be a Pawlak's approximation space. Then

$$\forall X \in 2^U (\underline{\varepsilon}(X) \subseteq X \subseteq \overline{\varepsilon}(X)),$$

that is, the maps $\underline{\varepsilon}$ and $\overline{\varepsilon}$ are contractive and extensive, respectively.

Corollary 3.10. $\underline{\varepsilon}(X) = X$ *if and only if* $X = \overline{\varepsilon}(X)$.

Proof. Since $\underline{\varepsilon}(X) \in \mathfrak{D}_{U/\varepsilon}$ ($\overline{\varepsilon}(X) \in \mathfrak{D}_{U/\varepsilon}$), then $X = \underline{\varepsilon}(X) \in \mathfrak{D}_{U/\varepsilon}$ ($X = \overline{\varepsilon}(X) \in \mathfrak{D}_{U/\varepsilon}$), and so, by Proposition 3.8, $X = \underline{\varepsilon}(X) = \overline{\varepsilon}(X)$ ($X = \overline{\varepsilon}(X) = \underline{\varepsilon}(X)$). □

Definition 3.11. Let (U, ε) be Pawlak's approximation space and $X \subseteq U$.
The ε-*boundary* of X is

$$B_\varepsilon(X) = \overline{\varepsilon}(X) \setminus \underline{\varepsilon}(X).$$

X is ε-*crisp*, if $B_\varepsilon(X) = \emptyset$, otherwise X is ε-*rough*.

Proposition 3.12 Let (U, ε) be Pawlak's approximation space and $X \subseteq U$.

1. X is ε-crisp if and only if X is ε-definable.

2. X is ε-rough if and only if X is ε-undefinable.

Proof. (1) ($\Rightarrow$) X is ε-crisp $\Leftrightarrow B_\varepsilon(X) = \overline{\varepsilon}(X) \setminus \underline{\varepsilon}(X) = \emptyset \Leftrightarrow \overline{\varepsilon}(X) \subseteq \underline{\varepsilon}(X)$. Proposition 3.9 implies $\underline{\varepsilon}(X) \subseteq \overline{\varepsilon}(X)$, and so $\underline{\varepsilon}(X) = \overline{\varepsilon}(X)$. According to Proposition 3.8, $\underline{\varepsilon}(X) = \overline{\varepsilon}(X) \Leftrightarrow X \in \mathfrak{D}_{U/\varepsilon}$.

($\Leftarrow$) Since $X \in \mathfrak{D}_{U/\varepsilon} \Leftrightarrow \underline{\varepsilon}(X) = \overline{\varepsilon}(X)$, so $B_\varepsilon(X) = \overline{\varepsilon}(X) \setminus \underline{\varepsilon}(X) = \emptyset$ trivially satisfies.

(2) It is the contrapositive version of (1). □

As a consequence of Proposition 3.12, the notions 'ε-crisp' and 'ε-definable' are synonymous to each other, and so are 'ε-rough' and 'ε-undefinable'.

4. Partial Approximation of Sets

Let U be any nonempty set called the *universe of discourse.*

Definition 4.1. Let $\mathfrak{B} \subseteq 2^U$ be a nonempty family of nonempty subsets of U called the *base system.* Its elements are the $\mathfrak{B}$-*sets.*

The family of sets $\mathfrak{D} \subseteq 2^U$ is $\mathfrak{B}$-*definable*, if its elements are $\mathfrak{B}$-sets, otherwise $\mathfrak{D}$ is $\mathfrak{B}$-*undefinable.*

A nonempty subset $X \in 2^U$ is $\mathfrak{B}$-*definable*, if there exists a $\mathfrak{B}$-definable family of sets $\mathfrak{D}$ such that $X = \bigcup \mathfrak{D}$, otherwise X is $\mathfrak{B}$-*undefinable.* The empty set is considered to be a $\mathfrak{B}$-definable set.

Let $\mathfrak{D}_{\mathfrak{B}}$ denote the family of $\mathfrak{B}$-definable sets of U.

Definition 4.2. Let $\mathfrak{B} \subseteq 2^U$ be a base system and X be any subset of U.

The *weak lower* $\mathfrak{B}$-*approximation* of X is

$$\mathfrak{C}^{\flat}_{\mathfrak{B}}(X) = \bigcup \{Y \mid Y \in \mathfrak{B}, Y \subseteq X\}, \tag{4.1}$$

and the *weak upper* $\mathfrak{B}$-*approximation* of X is

$$\mathfrak{C}^{\sharp}_{\mathfrak{B}}(X) = \bigcup \{Y \mid Y \in \mathfrak{B}, Y \cap X \neq \emptyset\}. \tag{4.2}$$

Clearly, $\mathfrak{C}^{\flat}_{\mathfrak{B}}(X), \mathfrak{C}^{\sharp}_{\mathfrak{B}}(X) \in \mathfrak{D}_{\mathfrak{B}}$, and the maps $\mathfrak{C}^{\flat}_{\mathfrak{B}}$, $\mathfrak{C}^{\sharp}_{\mathfrak{B}}$ are total, onto, and, in general, many-to-one. Furthermore, both of them are monotone.

Proposition 4.3 ([3], Theorem 17) Let the fixed base system $\mathfrak{B} \subseteq 2^U$ be given.

1. $\forall X \in 2^U (\mathfrak{C}^{\flat}_{\mathfrak{B}}(X) \subseteq \mathfrak{C}^{\sharp}_{\mathfrak{B}}(X))$.
2. $\forall X \in 2^U (\mathfrak{C}^{\flat}_{\mathfrak{B}}(X) \subseteq X)$—that is, $\mathfrak{C}^{\flat}_{\mathfrak{B}}$ is contractive.
3. $\forall X \in 2^U (X \subseteq \mathfrak{C}^{\sharp}_{\mathfrak{B}}(X))$ if and only if $\bigcup \mathfrak{B} = U$—that is, $\mathfrak{C}^{\sharp}_{\mathfrak{B}}$ is extensive if and only if $\mathfrak{B}$ covers the universe.

Proposition 4.4 ([3], Theorem 19) Let $\mathfrak{B} \subseteq 2^U$ be a base system. Then

1. $X \in \mathfrak{D}_{\mathfrak{B}}$ if and only if $\mathfrak{C}^{\flat}_{\mathfrak{B}}(X) = X$.
2. $X \notin \mathfrak{D}_{\mathfrak{B}}$ if and only if $\mathfrak{C}^{\flat}_{\mathfrak{B}}(X) \neq X$.

Unlike Pawlak's approximation spaces (cf. Proposition 3.8), by Proposition 4.4, the $\mathfrak{B}$-definable property is generally not equivalent to $\mathfrak{C}^{\flat}_{\mathfrak{B}}(X) = \mathfrak{C}^{\sharp}_{\mathfrak{B}}(X)$.

Definition 4.5. Let $\mathfrak{B} \subseteq 2^U$ be a base system and X be any subset of U.

The $\mathfrak{B}$-*boundary* of X is

$$\mathfrak{N}_{\mathfrak{B}}(X) = \mathfrak{C}^{\sharp}_{\mathfrak{B}}(X) \setminus \mathfrak{C}^{\flat}_{\mathfrak{B}}(X).$$

X is $\mathfrak{B}$-*approximatable* if $X \subseteq \mathfrak{C}^{\sharp}_{\mathfrak{B}}(X)$, otherwise it is said that X has a $\mathfrak{B}$-*approximation gap*.

Provided that $X \in 2^U$ is $\mathfrak{B}$-approximatable, X is $\mathfrak{B}$-*crisp*, if $\mathfrak{N}_{\mathfrak{B}}(X) = \emptyset$, otherwise is $\mathfrak{B}$-*rough*.

In general, $\mathfrak{N}_{\mathfrak{B}}(X) \notin \mathfrak{D}_{\mathfrak{B}}$, i.e., $\mathfrak{B}$-boundaries are usually $\mathfrak{B}$-undefinable.

A $\mathfrak{B}$-approximation gap calls our attention that the available knowledge about the system encoded in $\mathfrak{B}$ is not enough to approximate X. However, it may be natural or not.

According to Proposition 4.4, point (1), X is $\mathfrak{B}$-definable if and only if $X = \mathfrak{C}^{\flat}_{\mathfrak{B}}(X)$. If $X = \mathfrak{C}^{\flat}_{\mathfrak{B}}(X)$, then $X = \mathfrak{C}^{\flat}_{\mathfrak{B}}(X) \subseteq \mathfrak{C}^{\sharp}_{\mathfrak{B}}(X)$. However, it can easily be seen that $X = \mathfrak{C}^{\flat}_{\mathfrak{B}}(X)$ generally does not imply $X = \mathfrak{C}^{\sharp}_{\mathfrak{B}}(X)$. Hence, the notion '$\mathfrak{B}$-definable' does not imply the notion '$\mathfrak{B}$-crisp'. Thus, unlike Pawlak's approximation spaces (cf. Proposition 3.12), the notions '$\mathfrak{B}$-crisp' and '$\mathfrak{B}$-definable' are not synonymous to each other.

Possible interpretations of lower and upper $\mathfrak{B}$-approximations are the following [16, 18]:

- $\mathfrak{C}^{\flat}_{\mathfrak{B}}(X)$ is the set of all elements in U which can be *certainly* classified in a way that they belong to X with respect to $\mathfrak{B}$ ($\mathfrak{B}$-*positive region* of X).
- $\mathfrak{C}^{\sharp}_{\mathfrak{B}}(X)$ is the set of all elements in U which can be *possibly* classified in a way that they belong to X with respect to $\mathfrak{B}$.
- $U \setminus \mathfrak{C}^{\sharp}_{\mathfrak{B}}(X)$ is the set of all elements in U which can be *certainly* classified in a way that they *do not belong* to X with respect to $\mathfrak{B}$ ($\mathfrak{B}$-*negative region* of X).
- The elements in $\mathfrak{C}^{\sharp}_{\mathfrak{B}}(X) \setminus \mathfrak{C}^{\flat}_{\mathfrak{B}}(X)$ are *abstained* because they cannot be uniquely classified either as belonging to X or as not belonging to X with respect to $\mathfrak{B}$ ($\mathfrak{B}$-*borderline region* of X).

Notice that if $\bigcup \mathfrak{B} \neq U$, then $\forall X \subseteq U \setminus \bigcup \mathfrak{B}\ \forall B \in \mathfrak{B}(X \cap B = \emptyset)$. Consequently, for all these subsets $\mathfrak{C}^{\sharp}_{\mathfrak{B}}(X) = \bigcup \emptyset = \emptyset$, i.e., the empty set is the weak upper $\mathfrak{B}$-approximation of certain nonempty subsets of U.

Such cases may be excluded by a partial map, a so-called *strong upper* $\mathfrak{B}$-*approximation*. For more details, see [2, 3, 4, 5].

5. Conclusions and Future Work

In this paper we have presented a generalization of the rough set theory. Most notions of Pawlak's classical approximation spaces constitute compound ones which, however, split in two or more parts in our approach. This new approach helps us to understand the state of the compound nature of these notions and to specify their constituents.

The next most important task is to work out a partial approximative information system model by analogy with Pawlak's one [16] on which practical implementations of our theoretical model can be built up.

Finally, it must be mentioned that both Pawlak's rough set theory and our approach can extensively be applied in medical informatics, see, e.g., [7, 8, 9, 14].

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