

Biological Pest Control Based on Tensor Product Transformation Method

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Abstract: Pest management, based on biological control, has drawn attention from several research groups, due to the exclusion of chemical pesticides, which have debilitating outcomes, both on the environment and human health. Biological pest control policies have been determined using the model-based control approach. In this study, the tensor product model transformation (TPMT) was applied to model the nonlinear dynamic of the biological pest control system. Consequently, the feedback control law representing the biological pest control policy was synthesized based on LMI. Under the designed controller, the pest population was regulated based on the desired level. The simulation of the biological pest control system was presented to confirm the performance of the designed control law. It is evident, that the feedback control method based on TPMT can be employed appropriately, in this application.

Keywords: tensor product model transformation method; biological control; pest management; feedback control

1 Introduction

Based on the biological control, pest population was regulated to the desired level through the use of natural enemies, for example, pathogens, parasitoids, predators, antagonists, etc. [1-3]. The biological control can be implemented by three main approaches such as conservation, introduction and augmentation of natural enemies [1-4]. The conservation of natural enemies is applied when the local natural enemies exist in the target area. The introduction approach is employed when local natural enemies cannot handle invasions of nonnative pests. The augmentation is implemented through either the inoculative or the inundation releases depending on the effectiveness in reproducing the augmented natural enemies [1-4]. Several studies and applications of the biological pest control can be found in [1-6].

In general, the dynamic of biological systems can be described by the deterministic models representing the dynamic of ecosystems and epidemic systems as Lotka-Volterra model and compartmental models, respectively [4, 6-12]. The Lotka-Volterra model is a suitable model for the case of applying natural enemies to control pest population by considering pests as preys and natural enemies as predators.

Using Lotka-Volterra models brings about significant benefits for the attempts to further conduct the dynamic analysis and to determine control policies for regulating or maintaining pest population to be within the economic injury level (EIL) [4, 6, 7-17]. Typically, determination of the biological control policy based on mathematical models can be carried out by using optimal control according to Pontryagin's maximum principle [6, 13-14]. Applications of optimal control for pest management are presented in [6, 13-14]. Rafikov *et al.* [13-14] employed the Pontryagin's maximum principle to determine the biological control policy for the ecosystem represented by the n -dimensional Lotka-Volterra model.

Alternatively, the nonlinear feedback controller design can be utilized for defining the control policies for predator and prey systems [4, 15-18]. In [18], Meza *et al.* studied various nonlinear feedback control designs for the one-predator one-prey Lotka-Volterra system. Rafikov *et al.* [4] developed the mathematical model representing the relationship between a group of pests and a group of predators for the biological pest control system. In their work, the control policy was determined based on the LQR controller. Peubla *et al.* [15] applied the sliding mode control to define the control policy for a general pest control system with the case study of a one-pest and one-predator model. Recently, Peubla *et al.* [16] employed the modelling error compensation (MEC) method for the biological pest control for a class of the biological control system. Boonyaprapasorn *et al.* [17] determined the biological pest control based on the synergetic controller design. According to these previous works, the determination of control policies based on a nonlinear feedback control framework for biological processes is feasible, especially, when the system is in the strict feedback form with single input and

single output. For the case of multiple species of predator and prey systems, with multiple inputs and multiple outputs, defining control policies under the nonlinear feedback control framework may be complicated. Moreover, the design of control law of nonlinear systems is more complicated than that of the linear ones since the standard method of analysis is not available. Complication of synthesis feedback control framework depends on the structure and characteristics of the nonlinear system, for example, control objective, system order, dimensions of input and outputs, etc. [19-20].

Given the aforementioned complexities, the determination of the control policies under a linear feedback controller design framework is promising compared to existing approaches. One feasible and effective way to approximate these nonlinear ecosystem systems is the tensor product transformation model (TPMT) method. Introduced by Baranyi [21-27], TPMT promised an effective numerical approximation of the nonlinear dynamical system using a convex combination of linear time invariant (LTI) systems. Furthermore, during the transformation process, the high order singular decomposition (HOSVD) is performed such that LMI controller design tool can be executable simultaneously. With the parallel distributed compensation, control design according to LMI can be conducted efficiently based on the TP model [21-27]. As summarized in [24, 26], TP model transformation offers several advantages. For instance, the transformation can be implemented either in the form of equations or soft computing. The numbers of components in the TP model can be defined optimally by balancing between complexity and approximation error. In addition, the order of components is determined based on their relevance. The weighting functions of the TP transformation model can be easily determined under required constraints. Additionally, in the case of the Pseudo TP model transformation, the transformation can be achieved based on predefined weighting functions. The TP model transformation can be used as well to obtain common weighting functions for a set of models.

TPMT has been studied and utilized in the modeling and control of dynamical systems [19, 21-57]. It promises several added benefits in the prevention of overloaded computation [50-51], alleviation of the limitation to apply TPMT caused by different dimensions of inputs and output variables [34], capacities to be implemented under unknown structure [24], improvement of convex hull manipulation to handle the sensitivity problem in LMI-based control design [46-53] etc. The applications of TPMT in various engineering fields can be found in previous works, such as mechanical engineering [19, 36-38], aerospace engineering [29, 31, 39-41, 48], electrical engineering [42-43], robotics [33, 44] and filter design [45].

The TPMT method has been used as well in the application of biological processes. In previous work by Kovács and Eigner [58], the convex polytopic model of diabetes mellitus was developed and presented based on the TPMT

approach. Recently, Kovács and Eigner [59] examined the modeling of the tumor growth using this approach.

Since the biological pest control system is a nonlinear control system in the form of Lotka-Volterra model, the system can be expressed as the quasi-linear parameter varying (qLPV) system. Thus, the feedback controller design for this system can be conducted based on the TPMT method [4, 14, 35, 40].

Given the aforementioned effectiveness of TPM-based controller, authors intend to present an alternative and simple approach for defining of biological pest control polies under the linear feedback control framework. Thus, the application of the TPMT feedback controller design scheme to define the biological control policy becomes the main focus of this current work. The considered biological system is represented by a set of Lotka-Volterra equations containing two-pest species and two-predator species.

This paper is organized as follows. In Section 2, the mathematical model of the biological pest control is presented. Then, the TPMT-based controller design aimed to define the biological control policy is described in Section 3. Section 4 presents the simulation results of the biological pest control system under the determined control policy. The last section provides the conclusion of this study.

2 Mathematical Model of Biological Pest Control

According to [4, 14], the mathematical model of the biological pest control system was developed and represented by Lotka-Volterra system. The Lotka-Volterra model containing two pest species and two natural species is considered in this study. This model is used to describe the interaction between two parasitoids and two caterpillar species.

The growth rates of pest and natural enemy populations depend on the relationship between predators and preys and the biological control as presented in (1) [4, 14]:

$$\left. \begin{aligned} \dot{x}_1 &= x_1(r_1 - a_{11}x_1 - a_{12}x_2 - a_{13}x_3 - a_{14}x_4) \\ \dot{x}_2 &= x_2(r_2 - a_{21}x_1 - a_{22}x_2 - a_{23}x_3 - a_{24}x_4) \\ \dot{x}_3 &= x_3(-r_3 + a_{31}x_1 + a_{32}x_2) + U_1 \\ \dot{x}_4 &= x_4(-r_4 + a_{41}x_1 + a_{42}x_2) + U_2 \end{aligned} \right\}, \quad (1)$$

where x_1 and x_2 represent the population densities of the first and second caterpillars. The population densities of parasitoids are denoted by x_3 and x_4 respectively. The control inputs of the biological control policy corresponding to the growth rate of each parasitoid are defined by U_1 and U_2 . The reproduction rate or mortality rate of the i^{th} species is denoted by r_i , for $i = 1, \dots, 4$ and the

coefficient a_{ij} represents the rate corresponding to predation, competition and conservation from the interaction between the i^{th} species and the j^{th} species for $i, j = 1, \dots, 4$.

The differences between the state variables and control inputs to their corresponding steady values are defined as $y_1 = x_1 - x_1^*$, $y_2 = x_2 - x_2^*$, $y_3 = x_3 - x_3^*$, $y_4 = x_4 - x_4^*$, $u_1 = U_1 - u_1^*$ and $u_2 = U_2 - u_2^*$. The error system can be expressed as (2):

$$\dot{\mathbf{y}} = \bar{\mathbf{A}}\mathbf{y} + \mathbf{h}(\mathbf{y}) + \mathbf{B}\mathbf{u}, \quad (2)$$

where $\mathbf{y} = [y_1 \ y_2 \ y_3 \ y_4]^T$, $\mathbf{u} = [u_1 \ u_2]^T$,

$$\bar{\mathbf{A}} = \begin{bmatrix} r_1 - \sum_{j=1}^4 a_{1j}x_j^* - a_{11}x_1^* & -a_{12}x_1^* & -a_{13}x_1^* & -a_{14}x_1^* \\ -a_{21}x_2^* & r_2 - \sum_{j=1}^4 a_{2j}x_j^* - a_{22}x_2^* & -a_{23}x_2^* & -a_{24}x_2^* \\ -a_{31}x_3^* & -a_{32}x_3^* & r_3 - \sum_{j=1}^4 a_{3j}x_j^* & 0 \\ -a_{41}x_4^* & -a_{42}x_4^* & 0 & r_4 - \sum_{j=1}^4 a_{4j}x_j^* \end{bmatrix},$$

$$\mathbf{h}(\mathbf{y}) = \begin{bmatrix} -\sum_{j=1}^4 a_{1j}y_1y_j \\ -\sum_{j=1}^4 a_{2j}y_2y_j \\ -\sum_{j=1}^2 a_{3j}y_4y_j \\ -\sum_{j=1}^2 a_{4j}y_4y_j \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

In accordance with [4], the ecosystem (1) can be formulated in the form which is suitable to employ the TPMT. First, the desired steady state of, x_1 , x_2 , x_3 and x_4 are denoted as x_1^* , x_2^* , x_3^* and x_4^* . The control inputs corresponding to steady state of the system (1) are defined by u_1^* and u_2^* . Here, x_1^* and x_2^* are defined to be below the economic injury level. Then, the values of the corresponding x_3^* , x_4^* , u_1^* and u_2^* can be determined from (3):

$$\left. \begin{aligned} x_1^* (r_1 - a_{11}x_1^* - a_{12}x_2^* - a_{13}x_3^* - a_{14}x_4^*) &= 0 \\ x_2^* (r_2 - a_{21}x_1^* - a_{22}x_2^* - a_{23}x_3^* - a_{24}x_4^*) &= 0 \\ x_3^* (-r_3 + a_{31}x_1^* + a_{32}x_2^*) + u_1^* &= 0 \\ x_4^* (-r_4 + a_{41}x_1^* + a_{42}x_2^*) + u_2^* &= 0 \end{aligned} \right\}. \quad (3)$$

Readers can find more information about the mathematical model from previous works by Rafikov *et al.* [4] and Molter and Rafikov [14].

3 Tensor Product Transformation-based Controller Design of Biological Pest Control

In this section, the details about the application of the TPMT-based feedback control are provided. First, conversion of the biological pest control system by using the TPMT is presented in Section 3.1. Then, the detail of feedback controller design is explained in Section 3.2.

3.1 Tensor Product Model Transformation of Biological Pest Control System

According to [21-23, 25-26, 31], the state error corresponding to the ecosystem in (2) can be transformed to the convex tensor product (TP) model, which can be conducted as follows.

First, the dynamic error in (2) can be represented in the form of quasi-linear time varying parameters (qLPV) constructed as (4):

$$\begin{aligned} \dot{\mathbf{y}} &= \mathbf{A}(\mathbf{p}(t))\mathbf{y} + \mathbf{B}\mathbf{u} \\ &= \begin{bmatrix} \mathbf{A}(\mathbf{p}(t)) & \mathbf{B}(\mathbf{p}(t)) \end{bmatrix} \begin{bmatrix} \mathbf{y} \\ \mathbf{u} \end{bmatrix} \\ \dot{\mathbf{y}} &= \mathbf{S}(\mathbf{p}(t)) \begin{bmatrix} \mathbf{y} \\ \mathbf{u} \end{bmatrix}, \end{aligned} \quad (4)$$

where $\mathbf{S}(\mathbf{p}(t)) = \begin{bmatrix} \mathbf{A}(\mathbf{p}(t)) & \mathbf{B}(\mathbf{p}(t)) \end{bmatrix}$ is the system matrix. Considering $\bar{\mathbf{A}}$ and $\bar{\mathbf{h}}$ in (2), the component of the matrix \mathbf{A} can be selected as (5):

$$A_{11} = (r_1 - \sum_{j=1}^4 a_{1j}x_j^* - a_{11}x_1^*) - a_{11}p_1, \quad A_{12} = -a_{12}x_1^* - a_{12}p_1$$

$$\begin{aligned}
A_{13} &= -a_{13}x_1^* - a_{13}p_1, A_{14} = -a_{14}x_1^* - a_{14}p_1, A_{21} = -a_{21}x_2^* - a_{21}p_2 \\
A_{22} &= (r_2 - \sum_{j=1}^4 a_{2j}x_j^* - a_{22}x_2^*) - a_{22}p_2, A_{23} = -a_{23}x_2^* - a_{23}p_2 \\
A_{24} &= -a_{24}x_2^* - a_{24}p_2, A_{31} = -a_{31}x_3^*, A_{32} = -a_{32}x_3^* \\
A_{33} &= r_3 - \sum_{j=1}^4 a_{3j}x_j^* - (a_{31}p_1 + a_{32}p_2), A_{34} = 0, A_{41} = -a_{41}x_4^* \\
A_{42} &= -a_{42}x_4^*, A_{43} = 0, A_{44} = r_4 - \sum_{j=1}^4 a_{4j}x_j^* - (a_{41}p_1 + a_{42}p_2), \quad (5)
\end{aligned}$$

where $p_1(t) = y_1(t)$ and $p_2(t) = y_2(t)$

Then, the TPMT is applied to the qLPV system in (4). The convex combination of R constant linear time invariant (LTI) system representing the qLPV system can be obtained as (6):

$$\mathbf{S}(\mathbf{p}(t)) = \sum_{r=1}^R \omega_r(\mathbf{p}(t)) \mathbf{S}_r, \quad (6)$$

where the weighting function is defined by $\omega_r(\mathbf{p}(t))$ and \mathbf{S}_r is the r^{th} linear time invariant system denoted as (7):

$$\mathbf{S}_r = [\mathbf{A}_r \quad \mathbf{B}_r], r = 1, 2, \dots, R. \quad (7)$$

Therefore, the system in (4) can be expressed as (8):

$$\dot{\mathbf{y}} = \sum_{r=1}^R \omega_r(\mathbf{p}(t)) (\mathbf{A}_r + \mathbf{B}_r \mathbf{u}). \quad (8)$$

3.2 Controller Design

According to [4, 11], the objective control is to regulate the population of the pests to the desired level corresponding to the economic injury level (EIL). Thus, the control inputs need to be designed such that the system in (2) is stable.

The system in (4) is transformed into the TP model in (8). Then, the controller design can be performed according to a parallel distributed compensation (PDC) controller design frame work. The feedback controller is determined as (9):

$$\mathbf{u} = - \left(\sum_{r=1}^R \omega_r(\mathbf{p}(t)) \mathbf{K}_r \right) \mathbf{y}. \quad (9)$$

The gain matrices, \mathbf{K}_r , can be determined by solving the LMIs as shown in (10) and (11) [21, 25]:

$$-\mathbf{Y}\mathbf{A}_r^T - \mathbf{A}_r\mathbf{Y} + \mathbf{M}_r^T\mathbf{B}_r^T + \mathbf{B}_r\mathbf{M}_r > 0 \text{ for } r=1,\dots,R \quad (10)$$

and

$$\begin{aligned} &-\mathbf{Y}\mathbf{A}_r^T - \mathbf{A}_r\mathbf{Y} - \mathbf{Y}\mathbf{A}_s^T - \mathbf{A}_s\mathbf{Y} \\ &+ \mathbf{M}_s^T\mathbf{B}_r^T + \mathbf{B}_r\mathbf{M}_s + \mathbf{M}_r^T\mathbf{B}_s^T + \mathbf{B}_s\mathbf{M}_r \geq 0 \end{aligned} \text{ for } r \leq s \leq R, \quad (11)$$

where \mathbf{K}_r is defined as $\mathbf{K}_r = \mathbf{M}_r\mathbf{Y}^{-1}$. Under the control law in (7), the control system (4) can be stabilized asymptotically. Further details about the TPMT can be found in [21-23, 25-26, 31]. The TP tool program [23] can be employed to determine the TPMT and the feedback control in (9).

4 Simulations

The simulation results of the ecosystem representing the biological pest control system manipulated by the designed control law from Section 3 are presented and discussed. Here, the system parameters of (1) were from the previous works [4, 14] as follows: (i) $r_1 = 0.17$, $r_2 = 0.17$, $r_3 = 0.119$, $r_4 = 0.119$, (ii) $a_{11} = 0.00017$, $a_{12} = 0.00017$, $a_{13} = 0.0017$, $a_{21} = 0.000255$, $a_{22} = 0.00017$, $a_{23} = 0.00017$, $a_{24} = 0.0017$, $a_{31} = 0.00085$, $a_{32} = 0.000085$, $a_{41} = 0.00425$, $a_{42} = 0.00425$. The desired values of x_1^* and x_2^* were defined as $x_1^* = 9$ and $x_2^* = 9$, which corresponded to the economic injury level. From (3), it yielded $x_3^* = 0.5$, $x_4^* = 97.7$, $u_1^* = 0.0553$ and $u_2^* = 4.1522$. The spaces of $p_1(t) = y_1(t)$ and $p_2(t) = y_2(t)$ were selected as $[-5, 5]$ and $[-5, 5]$, respectively.

By using the TP tool program [23] with 100×100 sampling, the rank of the sampled tensor was found to be 2 on both dimensions. Thus, the system can be exactly represented by 4 vertex systems. Figures 1 and 2 display the obtained weighting functions. By solving the LMI conditions, it resulted in the following four linear feedback gains:

$$K_{1,1} = \begin{bmatrix} -13.6887 & 402.4572 & 1.1556 & -15.1487 \\ -257.5399 & 82.4338 & 11.7128 & 4.8434 \end{bmatrix}$$

$$K_{2,1} = \begin{bmatrix} -126.2939 & -2.7031 & 5.2014 & 4.1584 \\ 96.6865 & -219.2167 & -4.7439 & 4.8572 \end{bmatrix}$$

$$K_{1,2} = \begin{bmatrix} -31.1574 & -284.9870 & 0.8807 & 12.0729 \\ 250.5103 & -238.4877 & -11.5536 & 0.9592 \end{bmatrix}$$

$$K_{2,2} = \begin{bmatrix} -97.7995 & 422.5019 & 4.6144 & -13.3817 \\ -345.1646 & 256.9620 & 15.5817 & 1.2851 \end{bmatrix}.$$

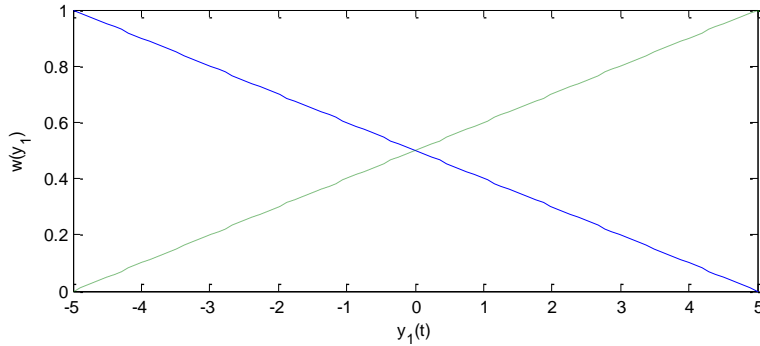


Figure 1
Weighting function on $p_1(t) = y_1(t)$

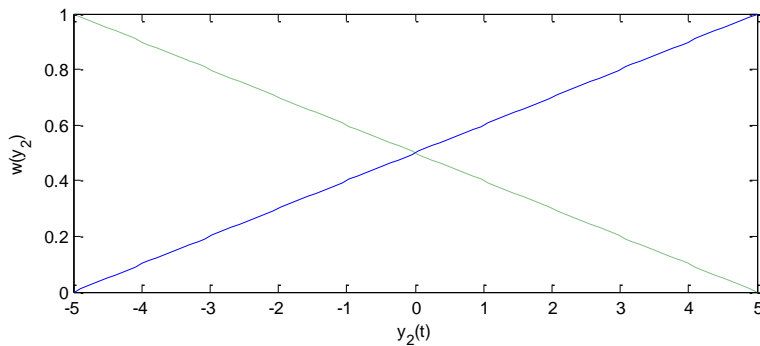


Figure 2
Weighting function on $p_2(t) = y_2(t)$

The time responses corresponding to the pests (caterpillars) and predators (parasitoids) of the biological control system as manipulated by the feedback control in (9) are presented in Figure 3. The plots of control inputs are shown in Figure 4. It was evident that the time responses of all pest converged to the desired levels as shown in Figure 3. Based on the synthesized control policy, the pest populations converged to the desired level which did not cause the economic damage.

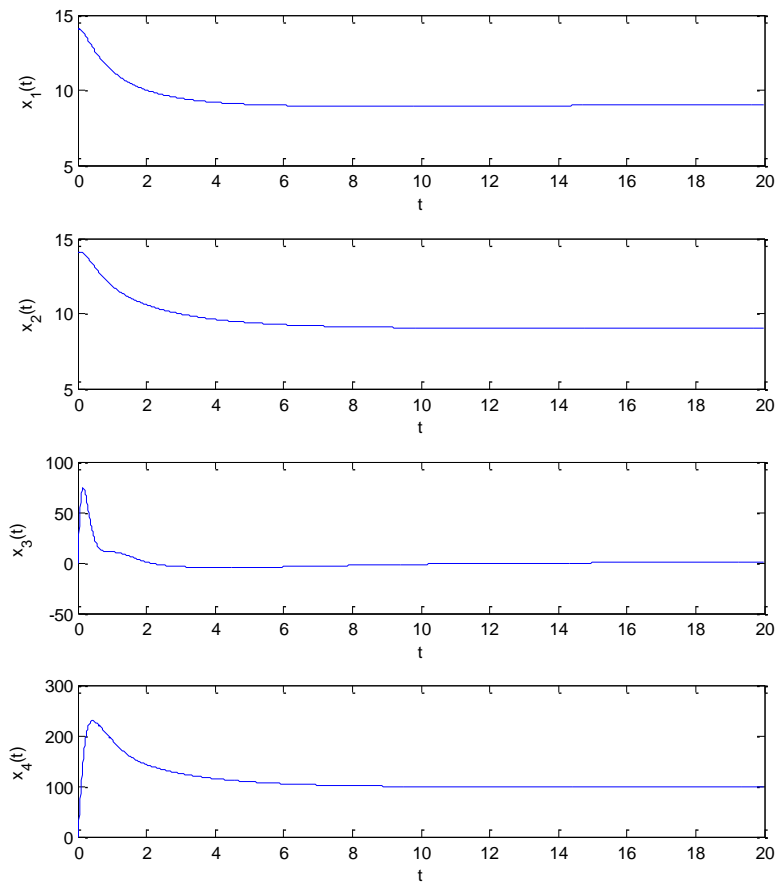


Figure 3
Time response of the Biological pest control system

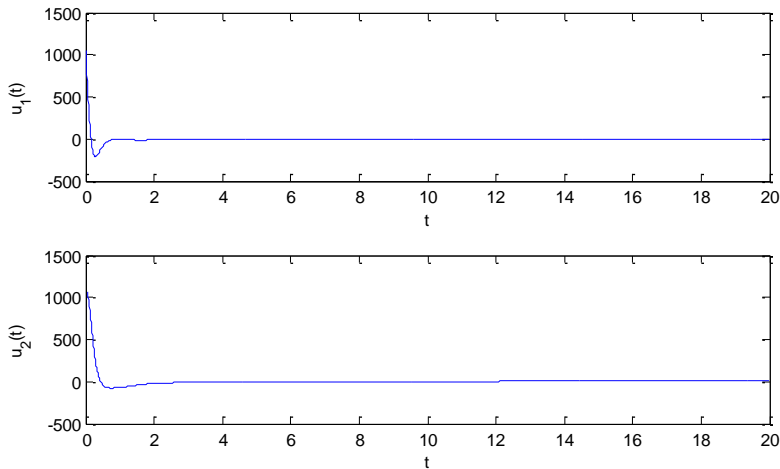


Figure 4
Biological control policy

Conclusion

From the current findings, the feedback controller design procedure, based on the tensor product model transformation (TPMT), was successfully employed to define control policy for the biological pest control system. According to the determined control policy, the pest populations were regulated at corresponding desired levels. The performance of the control method was confirmed by the simulation. Thus, the TPMT-based feedback control is recommended as an alternative method for determining control policy for biological pest control systems.

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