wir einen „contre coup", im Karpatenraum dagegen eine stabilere Tektonik der russischen Vorlandes und der Zwischengebirge. Die Tektonik wird durch die neogene *Mobilität der 2—3000 M. mächtigen jungen plastischen Sedimente* noch mehr verwickelt. Im weiteren zeigt sich ein rhythmisches Wiederholen der *Kontraktion und Dilatation*, was ein Charakteristikum unserer Mittelgebirge sein kann. Wir können Schuppung, Aufwölbung, Bruchfaltung, horizontale Blattverschiebung, Zerrung und Pressung kartieren, die untereinander in Zeit und Raum wechseln. Alldies spielt sich in *schmalen Zonen* ab, die sich durch stabile Treppen getrennt bewegen. Dies ist mit der innerkontinentalen Lage und der damit in Beziehung stehenden tektonischen Vielfältigkeit zu erklären. Hier würde eine tektonische Schematisierung nur ein Unterdrücken der wahrheitsgetreuen Beobachtungen bedeuten.

Die beiliegende Skizze soll einen allgemeinen Überblick geben und soll dennoch nicht als tektonische Karte betrachtet werden.*

Dr. Ferenc Szentes.

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AZ ANOMÁLIÁK MAGASSÁGI REDUKCIÓJÁRÓL

Irta: EGYED LÁSZLÓ

1—10 ábrával.

A Föld ideális alakja a nivószferoiddal írható le és elvileg ebben az esetben a Földet koncentrikus homogén héjakból képzeljük összetettnek, míg az ettől való eltérést a földkéreg inhomogénitásának tekintjük. Ennek megfelelően a nehézségi erőtér potenciálját éppenúgy, mint a nehézségi gyorsulást magát, két részre bonthatjuk: egy elméleti v. normál értékre és az anomália értékére.

Amennyiben az anomáliát figyelmen kívül hagyjuk, a különböző magasságokban észlelt gyorsulás értékeket úgy redukálhatjuk ugyanarra a szintre, pl. a tenger szintjére, hogy kivonjuk belőlük a free-air hatást és a topografikus hatással javított Bouguer-hatást. Ezt az eljárást nevezzük *magassági redukciónak.*

Az anomália értékét az észlelt értékből úgy kaphatjuk meg, hogy az észlelt értékre alkalmazzuk a magassági redukciót, ami az elméleti érték tengerszintre való redukciójának felel meg, — és a maradékból kivonjuk a tengerszintre megállapított elméleti értéket. Az így kapott anomália érték azonban nem a tengerszintre, hanem magára az észlelés helyére vonatkozik. Ezt az anomália értéket *Bouguer-anomáliának* nevezik. Amenynyiben az észlelési adatainkat egy nivófelületre akarjuk vonatkoztatni, meg kell oldanunk az anomáliák magassági redukcióját is és ezzel is redukálni értékeinket, egyébként az eddig használt Bouguer-anomália értéke azt jelenti, hogy észlelési adatainkat megszabadítottuk ugyan a Föld regionális hatásától, azonban vonatkozási felületül a Földnek valóságos felületét használtuk. Ez pedig bizonyos esetben komolyan tudja zavarni az anomáliákról alkotható képet.

Az anomáliák magassági redukciója belejátszik mindazokba a kérdésekbe, amelyek a földkéreg inhomogénitásával kapcsolatban merülnek fel, így pl. a geológiai kutatás kérdéseibe. De komolyan figyelmet érdemel az izosztázia kérdésénél is, hiszen éppen az izosztatikus szempontból érdekes helyeken a Bouguer-anomália tekintélyes, míg ugyanakkor a mérési adatok szintjei között több száz, esetleg több ezer méterek is szerepelhetnek.

E dolgozat keretén belül az anomáliák korrekciójának a kérdését oldjuk meg, beleértve a $\varDelta g$-korrekcióján kívül a gradiens és a görbület korrekcióját is.

Ha ismerjük a gradiens, vagy a görbület valamelyik komponensét — legyen ez pl. a gradiens északi komponense G_x — az észlelési helyen h-magasságban a tenger szintje felett, akkor a tenger szintjére vonatkozó anomália értékére egyszerű meggondolással a következőket kapjuk:

$$(G_x)_0 = (G_x)_h + \frac{1}{2}\left[\left(\frac{\partial G_x}{\partial z}\right)_0 + \left(\frac{\partial G_x}{\partial z}\right)_h\right] \cdot h.$$

Eszerint a gradiens és egészen analog módon a görbület anomália-értékének magassági redukciója megtörténhet, ha ismerjük a potenciálfüggvény harmadrendű deriváltjainak értékeit az észlelési helyen és a tenger szintjén.

A harmadrendű deriváltaknak torziósinga mérésekből való meghatározását sík terület esetében T. Olczak oldotta meg[1]. Ezeknek az eredményeknek az alkalmazását a kívánt szintekre vonatkozó harmadrendű deriváltak meghatározására lehetővé teszi egy fokozatos approximáció, amennyiben a mérési adatokat a felszínhez közeleső és geofizikai szempontból nem érdekes inhomogénitások nem zavarják.

A $\varDelta g$-anomália értéknek a tengerszintre való redukcióját a következő összefüggés adja:

$$\varDelta g_0 = \varDelta g_h + (G_z)_h \cdot h + \frac{1}{2}\left(\frac{\partial G_z}{\partial z}\right)_h \cdot h^2 + \cdots,$$

ha a valóságban rendesen elhanyagolható magasabbrendű tagokat nem vesszük figyelembe. A $\varDelta g$-értékek anomáliájának redukciója tehát a függőleges gradiens, valamint a másodrendű függőleges gradiens meghatározását kívánja. Ezeknek a mennyiségeknek a meghatározása már régi kérdése úgy a geofizikának, mint a geodéziának. A másodrendű függőleges gradiens meghatározása könnyen adódik az Olczak-féle eredményekből[3]. Ugyancsak könnyen adódik a függőleges gradiens változásának az értéke is sík terület esetében. Az egyetlen, de annál nehezebben megoldható probléma volt a függőleges gradiens abszolut nagyságának meghatározása. Ennek a kérdésnek megoldása jelenti főeredményünket. Vizsgálataink során ugyanis azt kaptuk, hogy ha két szomszédos, de különböző szinten fekvő két pontban graviméteres és torziósinga méréseket végzünk, akkor a graviméteres mérésekből adódó $\varDelta g$-változás értéke különbözni fog a gradiensekből Eötvös szerint számított $\varDelta g$-változás értékétől. Ha a két értéknek egymástól való eltérését elosztjuk a két észlelés szintkülönbségével, akkor éppen a két észlelési helyre vonatkozó függőleges gradiens középértékét kapjuk meg jó közelítéssel. A pontos összefüggést a következő formula szolgáltatja:

$$\frac{G_z^{I} + G_z^{II}}{2} = \frac{1}{z}\left[\delta\varDelta g - \delta\varDelta g_E + \frac{1}{12} \cdot D\right],$$

ahol $\delta\varDelta g$ jelenti a graviméteres észlelésekből származó $\varDelta g$-anomália változását, $\delta\varDelta g_E$ jelenti a gradiensekből Eötvös szerint számított $\varDelta g$-anomália változást, G_z jelöli a függőleges gradienst, míg z a szintkülönbséget; végül

$$D = \left(\frac{\partial G_x^{II}}{\partial x} - \frac{\partial G_x^{I}}{\partial x}\right) x^2 + 2\left(\frac{\partial G_x^{II}}{\partial y} - \frac{\partial G_x^{I}}{\partial y}\right) xy +$$

$$\left(\frac{\partial G_y^{II}}{\partial y} - \frac{\partial G_y^{I}}{\partial y}\right) y^2 + 2\left(\frac{\partial G_x^{II}}{\partial z} - \frac{\partial G_x^{I}}{\partial z}\right) xz + 2\left(\frac{\partial G_y^{II}}{\partial z} - \frac{\partial G_y^{I}}{\partial z}\right) yz + \left(\frac{\partial G_z^{II}}{\partial z} - \frac{\partial G_z^{I}}{\partial z}\right) z^2$$

egy korrekciós kifejezés, amely csak a harmadrendű deriváltak különbségével megszorzott másodrendű tagokat tartalmazza. A hozzá szükséges adatok meghatározása az Olczak-féle módszer említett általánosításával a torziósinga mérésekből könnyen adódik.

Ha három különböző szinten végzünk méréseket, akkor maga a függőleges gradiens értéke is kiadódik, mivel

$$G_z^{I} = \frac{1}{2}(G_z^{I} + G_z^{II}) + \frac{1}{2}(G_z^{I} + G_z^{III}) - \frac{1}{2}(G_z^{II} + G_z^{III})$$

Ha az így kapott értékeket a függőleges gradiensek változásaiból adódó értékekhez kapcsoljuk, akkor a terület minden pontjára levezethetjük a függőleges gradiens értékét, mivel sík területen bármely két észlelési pont között lévő függőleges gradiens változásnak az értékét, mint azt már említettük, meghatározhatjuk.

Eredményeinkből az is következik, hogy sík területen a graviméteres mérések mindenképen megbízhatóbb eredményt szolgáltatnak az izogammákat illetőleg, mint a torziósinga adatokból számított értékek.

Végül megemlítjük, hogy a függőleges gradiens értékének az ismerete hengeralakú függőleges hatók esetében mélységbecslésre is lehetőséget nyujt.

THE ELEVATION CORRECTION OF ANOMALIES

The determination of vertical gradient of anomalies by means of gravity meter and torsion balance measurements.)

BY L. EGYED*

The mathematical description of the gravity field is the simplest by the aid of the potential function of the field. The Earth's theoretical figure is determined by the niveauspheroid and in this case the supposition is made, that the Earth consists of concentric homogeneous shells, all deviations from this being considered as inhomogeneity of the Earth's crust. According to this supposition the gravity potential may be considered as result of two components, namely that of the theoretical value consisting of the effect of the homogeneous shells, and the value of anomalies originating from the inhomogeneities of the crust:

$$U = U_n + U_a.$$

As a consequence of this the value of gravity is

$$g = \frac{\partial U_n}{\partial z} + \frac{\partial U_a}{\partial z}$$

where the direction of z-axis coincides with the vertical. Here

$$\frac{\partial U_n}{\partial z} = \gamma_0 + 2\pi f.\sigma\, h - 0{\cdot}0003086.h - T$$

$$\gamma_0 = 978{\cdot}049.(1 + 0{\cdot}0052884.\sin^2\varphi - 0{\cdot}0000059.\sin^2 2\varphi).$$

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being the normal value of gravity on the sea level (in accordance with Cassinis' international formula, 1930), and

$$(2\pi f\sigma - 0{\cdot}0003086).h - T$$

is the total value of the effect of elevation.

This effect of elevation consists essentially of three components. The first component is the „free-air"-effect, originating from the fact, that not taking in account the effect of masses lying between the station and the sea level, the place of observation is with h meters farther from the Earth's center, than the sea level. The value of this is $-0{,}3086\,.\,h$ milligals.

The effect of the masses lying between the sea level and the station is taken into account by the Bouguer-value $2\pi f\sigma.h$. which corresponds to the attraction of an infinite horizontal plate, with the thickness h. The effect caused by the deviation of topography is considered by the topographical effect T.

Neglecting the anomaly, the observed gravity value $\frac{\partial U_n}{\partial z}$ may be reduced to a reference level f. e. to the sea level by substracting the effect of elevation from the observed value, i. e. the free-air and the Bouguer-effect, corrected with the topographical effect. This is the *elevation reduction.*

On account of these the value of anomaly may be determined by using the elevation reduction on the observed value, which corresponds to the reduction of the theoretical value on the sea level. The deviation of the obtained result from the normal value is the anomaly. The anomaly resulted on this way however refers not to the sea level, only to the place of observation. This anomaly is called the *Bouguer-anomaly.*

The fact, that the anomaly on the sea level differs from that on the level of observation, could be enlightened by the following exemple:

In the homogeneous Earth's crust corresponding to the ideal gravity let to be a homogeneous sphere G (see fig. 1.) with a density,

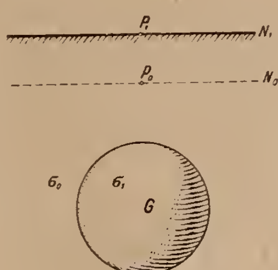


Fig. 1.

which differs from that of the Eartsh's crust, N_1 being the level of observation, N_0 the sea level. The value of gravity in P_1 is composed from the theoretical value g_1 and from the effect of attraction of the sphere with desinty $\sigma_1 - \sigma_0$, the so called anomaly $\left(\frac{\partial U_a}{\partial z}\right)_1$ On the sea level this value is $g_0 + \left(\frac{\partial U_a}{\partial z}\right)_0$, where g_0 may be deduced from g_1 by the elevation reduction.

But the theoretical elevation correction doesn't modify the value of $\left(\frac{\partial U_a}{\partial z}\right)_1$, which may differ from $\left(\frac{\partial U_a}{\partial z}\right)_0$ by a considerable amount, being the sphere with the density $\sigma_1 - \sigma_0$ in some case essentially nearer to P_0.

The value of anomaly as a function of elevation is shown in figures 2a, and 2b, where the figure *a* represents the effect of a slablike structure, infinite perpendicular to the design's plane and confined by two 1000 m high faults, the effect of this observed on the levels 200 m, 600 m and 1000 m above the top of the structure. This diagram demonstrates,

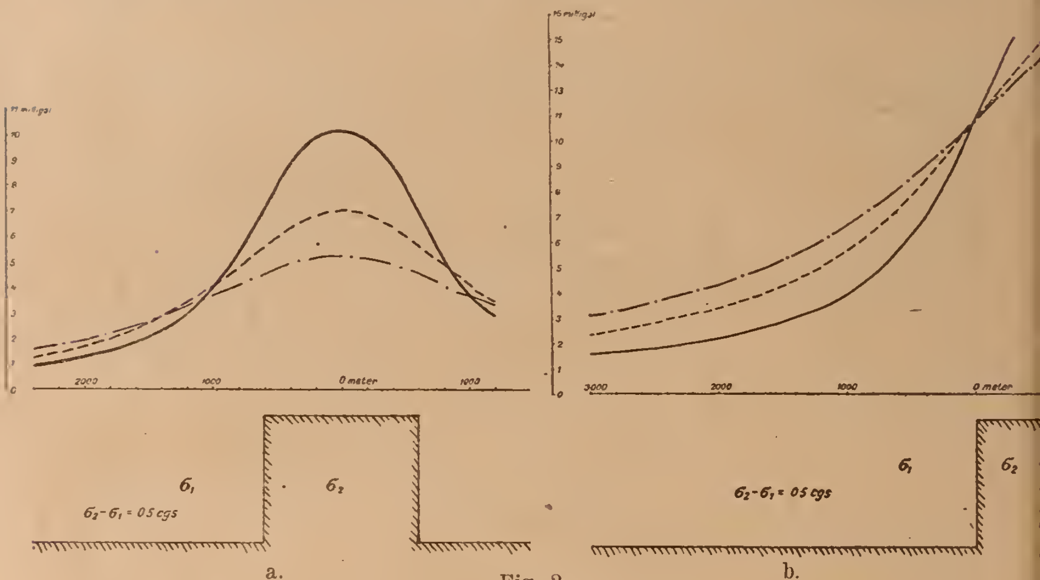


Fig. 2.

that the variation of the anomalies' value observed on different levels surpasse the 50% of the maximum value, and in consequence this is able to confuse the picture obtained from the structure, wich has caused the anomaly. The figure *b* represents the effect of a fault of the same magnitude in the mentioned levels.

Where the free-air and Bouguer corrections may be applied, the theoretical gradient and curvature values are independent of the elevation as a natural consequence of the fact that the effect of elevation is independent from the latitude also. But this is not true in the case of gradient and curvature of anomalies. The values of gradient and curvature of anomalies are varying with the elevation and the variation may be considerable, as it can be seen by the following example:

The value of gradient caused by a fault lying under the surface in a depth equal with its thickness, is $2\pi f.\sigma.0{,}6932$. This value at a level twice as high above the top of the fault is $2\pi f.\sigma.0{,}4055$. The difference between the two values is about 75 %.

It is not difficult to construct such an example also for the curvature.

These examples show clearly, that if the reference on the same level of all collected data is desired, then it is necessary to make an elevation reduction for anomalies also. Otherwise with this reduction our data are only eliberated from the Eart's theoretical effect, but as reference surface the Earth's real surface was used, and not the sea level.

These facts are important in all questions. which are in connection with the inhomogeneities of the Earth's crust f. e. in the geological surveys. But they are very important also in the theory of isostasy, considering that on the places interesting for isostasy the Bouguer-anomalies are great, while at the same time the amount of level differences in observations may attain some hundreds, or thousands of meters.

In the following will be discussed the elevation reductions of anomalies, including the correction of Δg-values and the reduction of gradient and curvature values also.

2. For easier convenience the following signs will be introduced:

The gradient components: $\frac{\partial^2 U}{\partial x \partial z} = G_x$ and $\frac{\partial^2 U}{\partial y \partial z} = G_y$

The curvature components: $\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2} = H_\Delta$ resp. $2\frac{\partial^2 U}{\partial x \partial y} = H_{xy}$

If the value of one component, f. e. G_x on the observation level is known, then the value of anomaly is on the sea level theoretically the following:

$$(G_x)_0 = (G_x)_h + \left(\frac{\partial G_x}{\partial z}\right)_h \cdot h + \frac{1}{2!}\left(\frac{\partial^2 G_x}{\partial z^2}\right)_h \cdot h^2 + \dots$$

In this series the terms of higher order as the second, may be practically neglected. But in this case this may be written:

$$(G_x)_0 = (G_x)_h + \frac{1}{2}\left(\frac{\partial G_x}{\partial z}\right)_h \cdot h + \frac{1}{2}\left[\left(\frac{\partial G_x}{\partial z}\right)_h + \left(\frac{\partial^2 G_x}{\partial z^2}\right)_h \cdot h\right] \cdot h$$

where:

$$\left(\frac{\partial G_x}{\partial z}\right)_h + \left(\frac{\partial^2 G_x}{\partial z^2}\right)_h \cdot h = \left(\frac{\partial G_x}{\partial z}\right)_0$$

and in consequence:

$$(G_x)_0 = (G_x)_h + \frac{1}{2}\left[\left(\frac{\partial G_x}{\partial z}\right)_0 + \left(\frac{\partial G_x}{\partial z}\right)_h\right] \cdot h$$

I. e. the reduction of the gradient value to the sea level — and in the same manner that of the curvature — is possible in that case, when the third order derivatives of the potential function are known at the site of observation and on the sea level.

The determination of the third order derivatives by torsion balance measurements in the case of flat areas is resolved by T. Olczak.[1]

[1] The first steps to determine the third order derivatives are taken already by Eötvös, (R. Eötvös: Bestimmung der Gradienten der Schwerkraft und ihrer Niveauflächen mit Hülfe der Drehwage. Abh. d. allg. Konf. d. Erdm. in Budapest, 1906.) The formulae derived are also experimentally demonstrated by him, observing with torsion balance in two different levels with an elevation difference of a few cm-s. The observed and calculated values agreed within the accuracy of observations.

The question of third order derivatives was examined in details by T. Olczak. (T. Olczak: The measurements with the Eötvös torsion balance and the problem of determining the higher normal derivatives of the external gravity potential. Polish, with an english abstract, [Inst. Géol. de Pologne. Bull. 45. Série Géoph. Warszawa, 1948.])

T. Olczak received for the place of observation the following formulae:[2]

$$\frac{\partial G_x}{\partial z} = \frac{\partial H_\Delta}{\partial x} - \frac{\partial H_{xy}}{\partial y}$$

$$\frac{\partial G_y}{\partial z} = -\frac{\partial H_{xy}}{\partial x} + \frac{\partial H_\Delta}{\partial y}$$

$$\frac{\partial H_\Delta}{\partial z} = -\frac{\partial G_x}{\partial x} + \frac{\partial G_y}{\partial y}$$

$$\frac{\partial H_{xy}}{\partial z} = 2\frac{\partial G_x}{\partial y} = 2\frac{\partial G_y}{\partial x}$$

Thus the third order derivatives in flat regions may be determined from the gradient and curvature sections obtained from torsion balance measurements executed in a dense net.

If the layers lying near to the surface are not disturbed by inhomogeneities uninterested from geological, geophysical, or geodesical point of view, the third order derivatives referred to the sea level or the level of observation may be determined by a successive approximation in the following way: it is supposed in the first approximation, that all observational data are in the same elevation, and on the ground of this supposition we compute the approximative values of third order derivatives. These approximative values render us possible to determine the approximative gradient and curvature components referring to the sea level by the aid of the formula:

$$(G)_0 = (G)_h + \frac{1}{2}\left[\left(\frac{\partial G}{\partial z}\right)_0 + \left(\frac{\partial G}{\partial z}\right)_h\right] \cdot h$$

in first approximation with the supposition, that:

$$\left(\frac{\partial G}{\partial z}\right)_0 = \left(\frac{\partial G}{\partial z}\right)_h$$

and on the same way the values referring to the level of observation. These new values are suitable for deducing the values $\left(\frac{\partial G}{\partial z}\right)_0$ and $\left(\frac{\partial G}{\partial z}\right)_h$ with a better approximation. We continue the approximation in this manner, till it has a practical sense.

3. The value of Δg-anomaly observed by the aid of gravity meters or pendulums is on the sea level theoretically:

$$\left(\frac{\partial U_a}{\partial z}\right)_0 = \left(\frac{\partial U_a}{\partial z}\right)_h + \left(\frac{\partial^2 U}{\partial z^2}\right)_h \cdot h + \frac{1}{2}\left(\frac{\partial^3 U}{\partial z^3}\right)_h \cdot h^2 + \cdots$$

Practically the terms of third and higher order may be mostly neglected. These are shown on the figures 3 a, b, c and 4 a, b, c, where the effects of the structures represented in the figures 2 are drawn in the ground levels lying 200—600—1000 m respectively above the top of the structures. The thicker lines represent the theoretical values, the thinner ones, that of calculated from the values on the level noted on the curves and with the supposition, that the terms higher as the second order are neglected. The density-difference between the structure and the sorrounding sediments is supposed to be 0,5 cgs. The differences between the theoretical and computed values as they are shown by the diagrams are of no importance.

[2] T. Olczak, l. c. p. 37.

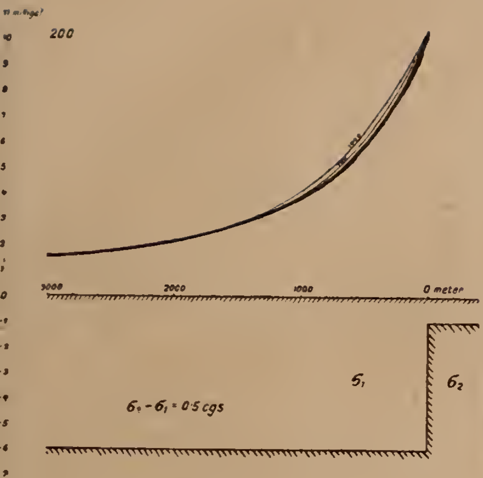


Fig 3 a.

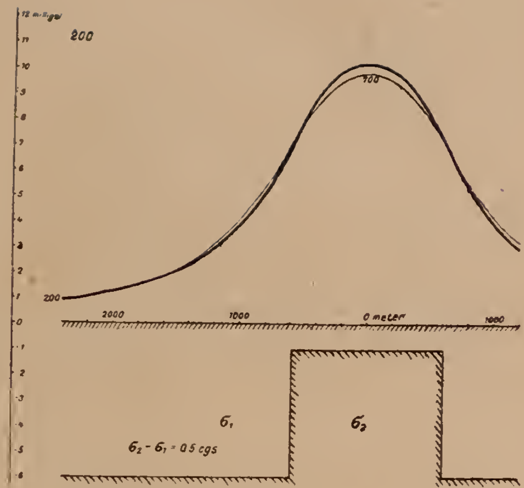


Fig. 4 a.

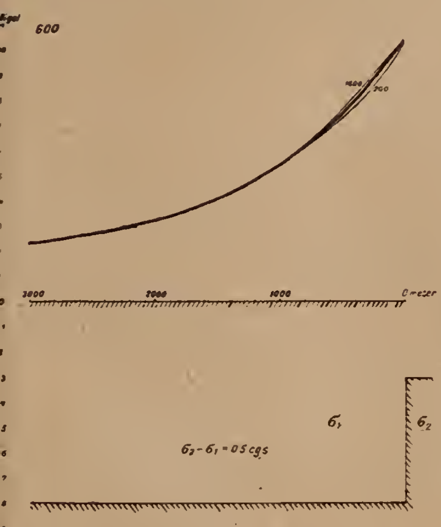


Fig. 3. b.

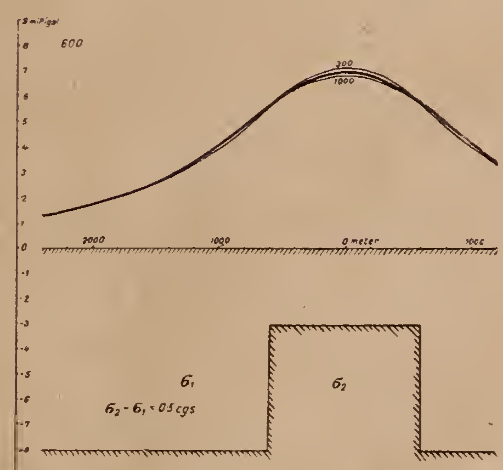


Fig. 4. b.

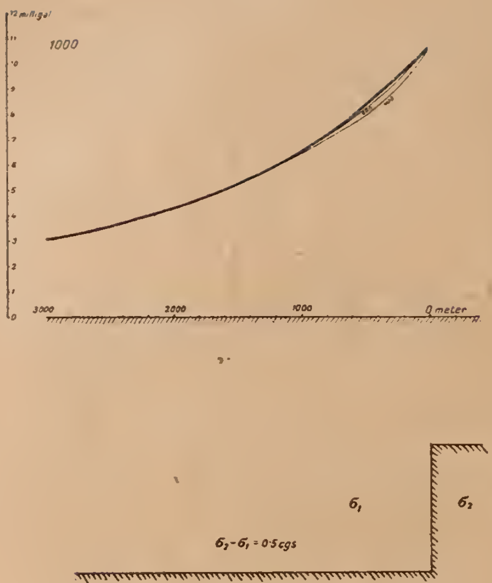


Fig. 3 c.

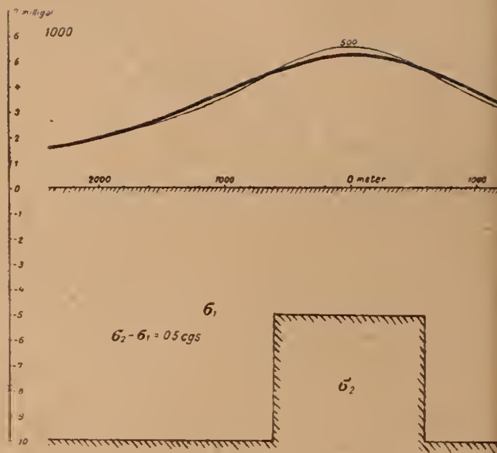


Fig. 4 c.

Thus the question of elevation reduction of Δg-values require the determination of the vertical gradient $G_z = \frac{\partial^2 U}{\partial z^2}$ and that of the $\frac{\partial G_z}{\partial z} = = \frac{\partial^3 U}{\partial z^3}$, which may be called the „vertical gradient of second order".

The values of vertical gradients of second order may be computed from the results of torsion balance measurements, using the derived Laplace equation. This is.[3]

$$\frac{\partial^3 U}{\partial z^3} = -\left(\frac{\partial G_x}{\partial_x} + \frac{\partial G_y}{\partial_y}\right)$$

where $\frac{\partial G_x}{\partial x}$ and $\frac{\partial G_y}{\partial y}$ may be deduced without difficulty from the sections of gradient values.

But in flat areas we can very easily obtain the change of vertical gradients between two points also by the formula.[4]

$$dG_z = \frac{\partial G_x}{\partial z} \cdot dx + \frac{\partial G_y}{\partial z} dy$$

or more generally by the finite formula:

$$G_z^{\mathrm{II}} - G_z^{\mathrm{I}} = \frac{1}{2}\left(\frac{\partial G_x^{\mathrm{I}}}{\partial z} + \frac{\partial G_x^{\mathrm{II}}}{\partial z}\right) \cdot x + \frac{1}{2}\left(\frac{\partial G_y^{\mathrm{I}}}{\partial z} + \frac{\partial G_y^{\mathrm{II}}}{\partial z}\right) \cdot y.$$

If similarly to the gradient components the values $\frac{\partial G_x}{\partial z}$ and $\frac{\partial G_y}{\partial z}$ are regarded as vector components, this vector may be called „gradient of

[3] T. Olczak. 1. c. p. 36.
[4] T. Olczak, 1. c. p. 38.

second order", then by the aid of these second order gradients the „isogams of second order" can be computed per analogiam of isogams calculated from the gradient values. If the values of vertical gradient G in some points of the area are known, connecting and adjusting to these the second order isogams, the values of vertical gradients to all points of the area can be obtained.

The question is only, how to determine the absolute value of vertical gradient of anomaly by the aid of gravity instrument now available, i. e. with the gravity meter and the torsion balance.

In the followings — and this is the main result of the present contribution — a method will be given for the determination of vertical gradient of anomaly by connecting the torsion balance and gravity meter observation in an area with different elevations.

4. In the points P_{I} and P_{II} (see fig. 5.) The following values are supposed to be known:

I. The Bouguer-anomaly determined by the gravity meter:

$$\Delta g = \frac{\partial U}{\partial z}.$$

II. The gradient components determined by the torsion balance:

$$G_x = \frac{\partial^2 U}{\partial x \partial z} \text{ and } G_y = \frac{\partial^2 U}{\partial y \partial z}.$$

III. The third order derivatives of the potential function determined by the aid of torsion balance measurements executed on the area:

$$U_{xxz} = \frac{\partial G_x}{\partial x}; \; U_{xyz} = \frac{\partial G_x}{\partial y} = \frac{\partial G_y}{\partial x}; \; U_{yyz} = \frac{\partial G_y}{\partial y}$$

$$U_{xzz} = \frac{\partial G_x}{\partial z}; \qquad U_{yzz} = \frac{\partial G_y}{\partial z}; \; U_{zzz} = \frac{\partial G_z}{\partial z}$$

which are connected with the formula:

$$\frac{\partial G_z}{\partial x} + \frac{\partial G_y}{\partial y} + \frac{\partial G_z}{\partial z} = 0$$

derived from the Laplace-equation.

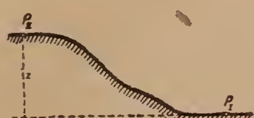


Fig. 5.

If the derivatives of the potential function are known in the point P_{I} inclusive the forth order terms, the value of Δg in the point P_{II} may be deduced by the aid of the Taylor series, using these derivatives in the following manner:

$$U_z^{\text{II}} = U_z^{\text{I}} + U_{xz}^{\text{I}} . x + U_{yz}^{\text{I}} . y + U_{zz}^{\text{I}} . z +$$

$$+ \frac{1}{2} (U_{xxz}^{\text{I}} . x^2 + 2 U_{xyz}^{\text{I}} xy + U_{yyz}^{\text{I}} . y^2 + 2 U_{xzz}^{\text{I}} xz + 2 U_{yzz}^{\text{I}} . yz + U_{zzz}^{\text{I}} . z^2) +$$

$$+ \frac{1}{6} (U_{xxxz}^{\text{I}} x^3 + 3 U_{xxyz}^{\text{I}} x^2 y + 3 U_{xyyz}^{\text{I}} xy^2 + U_{yyyz}^{\text{I}} . y^3 + 3 U_{xxzz}^{\text{I}} x^2 z +$$

$$+ 6 U_{xyzz}^{\text{I}} xyz + 3 U_{xzzz}^{\text{I}} . xz^2 + 3 U_{yyzz}^{\text{I}} y^2 z + 3 U_{yzzz}^{\text{I}} yz^2 + U_{zzzz}^{\text{I}} . z^3)$$

This formula by arranging the terms may be written also in the following form:

$$U_z^{II} - U_z^{I} =$$

$$= \frac{1}{2} \Big\{ U_{xz}^{I} + [U_{xz}^{I} + (U_{xxz}^{I}\, x + U_{xyz}^{I}\, y + U_{xzz}^{I}\, z) + \frac{1}{2}(U_{xxxz}^{I}\, x^2 + 2\, U_{xxyz}^{I}\, xy +$$

$$+ U_{xyyz}^{I}\, y^2 + 2\, U_{xxzz}^{I}\, xz + 2\, U_{xyzz}^{I}\, y.z + U_{xzzz}^{I}.z^2)] \Big\} \cdot x +$$

$$+ \frac{1}{2} \Big\{ U_{yz}^{I} + [U_{yz}^{I} + (U_{xyz}^{I}\, x + U_{yyz}^{I}\, y + U_{yzz}^{I}\, z) + \frac{1}{2}(U_{xxyz}^{I}\, x^2 + 2\, U_{xyyz}^{I}\, xy +$$

$$+ U_{yyyz}^{I}\, y^2 + 2\, U_{xyzz}^{I}\, xz + 2\, U_{yyzz}^{I}\; yz + U_{yzzz}^{I}.z^2)] \Big\} \cdot y +$$

$$+ \frac{1}{2} \Big\{ U_{zz}^{I} + [U_{zz}^{I} + (U_{xzz}^{I}\, x + U_{yzz}^{I}\, y + U_{zzz}^{I}.z) + \frac{1}{2}(U_{xxzz}^{I}\, x^2 + 2\, U_{xyzz}^{I}\, xy +$$

$$+ U_{yyzz}^{I}\, y^2 + 2\, U_{xzzz}^{I}\, xz + 2\, U_{yzzz}^{I}\; yz + U_{zzzz}^{I}.z^2)] \Big\} \cdot z -$$

$$- \frac{1}{12} \Big\{ [(U_{xxz}^{I} + U_{xxxz}^{I}\, x + U_{xxyz}^{I} y + U_{xxzz}^{I} z) - U_{xxz}^{I}]\, x^2 +$$

$$+ 2\, [(U_{xyz}^{I} + U_{xxyz}^{I} x + U_{xyyz}^{I} y + U_{xyzz}^{I} z) - U_{xyz}^{I}]\, xy +$$

$$+ \; [(U_{yyz}^{I} + U_{xyyz}^{I} x + U_{yyyz}^{I} y + U_{yyzz}^{I} z) - U_{yyz}^{I}]\, y^2 +$$

$$+ 2\, [(U_{xzz}^{I} + U_{xxzz}^{I} x + U_{xyzz}^{I} y + U_{xzzz}^{I} z) - U_{xzz}^{I}]\, xz +$$

$$+ 2\, [(U_{yzz}^{I} + U_{xyzz}^{I} x + U_{yyzz}^{I} y + U_{yzzz}^{I} z) - U_{yzz}^{I}]\, yz +$$

$$+ 2\, [(U_{zzz}^{I} + U_{xzzz}^{I}\, x + U_{yzzz}^{I}\, y + U_{zzzz}^{I} z) - U_{zzz}^{I}]\, z^2 \Big\}$$

But the components of the gradients are

$$U_{xz}^{II} = U_{xz}^{I} + (U_{xxz}^{I} x + U_{xyz}^{I} y + U_{xzz}^{I} z) + \frac{1}{2}(U_{xxxz}^{I} x^2 + \cdots)$$

$$U_{yz}^{II} = U_{yz}^{I} + (U_{xyz}^{I} x + U_{yyz}^{I} y + U_{yzz}^{I} z) + \frac{1}{2}(U_{xxyz}^{I} x^2 + \cdots)$$

$$U_{zz}^{II} = U_{zz}^{I} + (U_{xzz}^{I} x + U_{yzz}^{I} y + U_{zzz}^{I} z) + \frac{1}{2}(U_{xxzz}^{I} x^2 + \cdots)$$

and it is also

$$U_{xxz}^{II} = U_{xxz}^{I} + U_{xxxz}^{I} x + U_{xxyz}^{I} y + U_{xxzz}^{I} z$$

and the other expressions can be developed in similar way.

Using these and the former abbreviations, our equation becomes:

$$\overset{II}{\Delta g} - \overset{I}{\Delta g} = \frac{1}{2}(G_x^{I} + G_x^{II})\, x + \frac{1}{2}(G_y^{I} + G_y^{II})\, y + \frac{1}{2}(G_z^{I} + G_z^{II})\, z -$$

$$- \frac{1}{12} \Bigg[\left(\frac{\partial G_x^{II}}{\partial x} - \frac{\partial G_x^{I}}{\partial x} \right) x^2 + 2 \left(\frac{\partial G_x^{II}}{\partial y} - \frac{\partial G_x^{I}}{\partial y} \right) xy + \left(\frac{\partial G_y^{II}}{\partial y} - \frac{\partial G_y^{I}}{\partial y} \right) y^2 +$$

$$+ 2 \left(\frac{\partial G_x^{II}}{\partial z} - \frac{\partial G_x^{I}}{\partial z} \right) xz + 2 \left(\frac{\partial G_y^{II}}{\partial z} - \frac{\partial G_y^{I}}{\partial z} \right) yz + \left(\frac{\partial G_z^{II}}{\partial z} - \frac{\partial G_z^{I}}{\partial z} \right) . z^2 \Bigg].$$

The quantities occuring in this expression, except the value $\frac{1}{2}(G_z^{I} + G_z^{II})$ may be determined by gravity meter, or torsion balance measurements. Namely the value $\delta \Delta g = \overset{II}{\Delta g} - \overset{I}{\Delta g}$ is obtained by gravity meter measurements executed at the points P_I and P_{II}. The two gradient components in P_I and P_{II} : G_x^{I}, G_y^{I} resp. G_x^{II}, G_y^{II} are determined by torsion balance measurements.

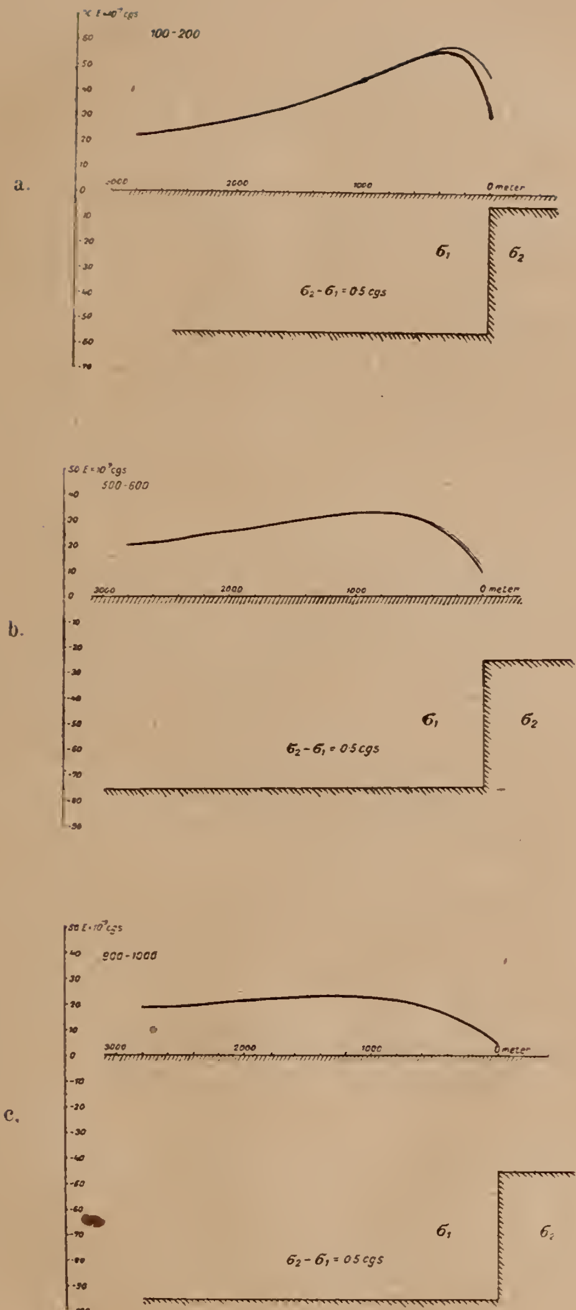
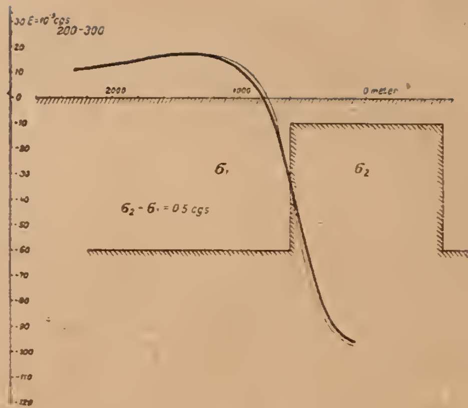
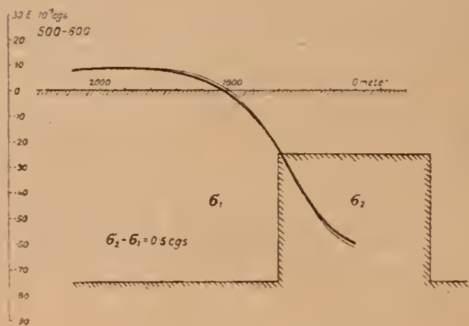

a.
100 - 200
2000
1000
0 meter
б₁
б₂
б₂ - б₁ = 0.5 cgs
b.
500 - 600
3000
2000
1000
0 meter
б₁
б₂
б₂ - б₁ = 0.5 cgs
c.
900 - 1000
3000
2000
1000
0 meter
б₁
б₂ - б₁ = 0.5 cgs

Fig. 6.

a.


200-300
2000
1000
0 meter
σ_1
σ_2
$\sigma_2 - \sigma_1 = 0{,}5$ cgs

b.


500-600
2000
1000
σ_1
σ_2
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c.

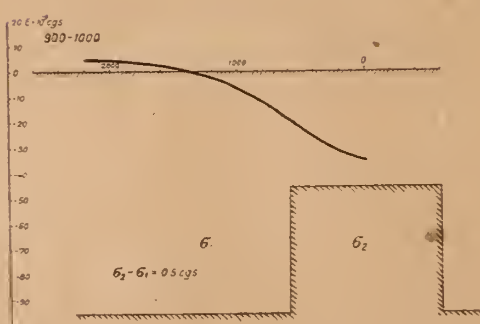

900-1000
1000
0
σ_2
$\sigma_2 - \sigma_1 = 0{,}5$ cgs

Fig. 7.

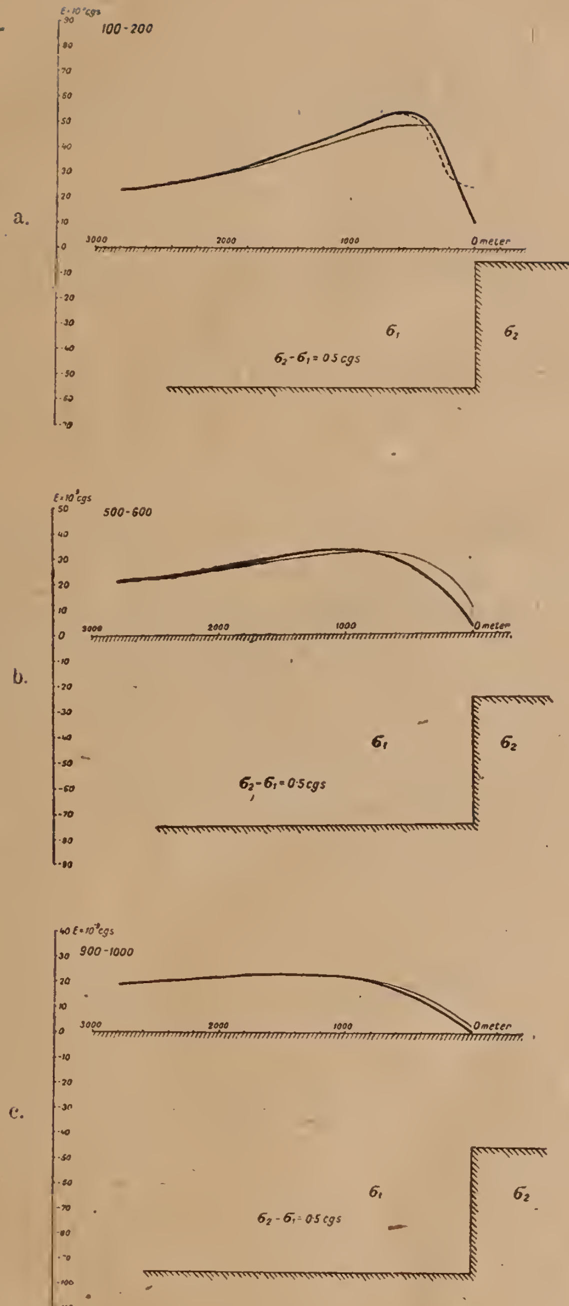
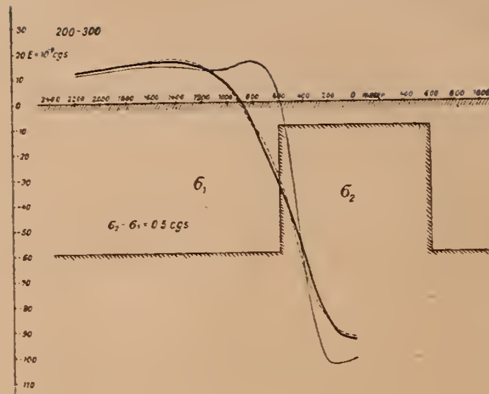
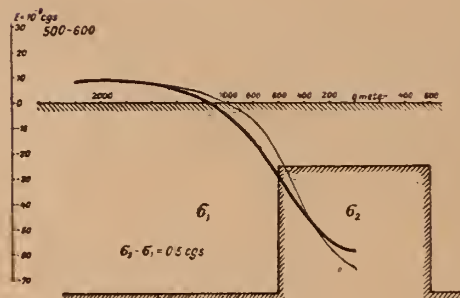

a.
100-200
3000
2000
1000
0 meter
σ_1
σ_2
$\sigma_2 - \sigma_1 = 0.5$ cgs
b.
500-600
3000
2000
1000
0 meter
σ_1
σ_2
$\sigma_2 - \sigma_1 = 0.5$ cgs
c.
900-1000
3000
2000
1000
0 meter
σ_1
σ_2
$\sigma_2 - \sigma_1 = 0.5$ cgs

Fig. 8.

a.


200-300
б1
б2
б2 - б1 = 0.5 cgs

b.


500-600
б1
б2
б2 - б1 = 0.5 cgs

t.

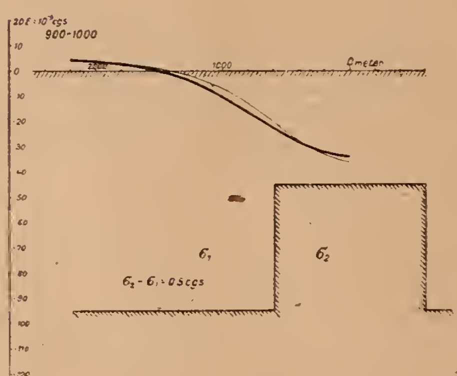

900-1000
б1
б2
б2 - б1 = 0.5 cgs

Fig. 9.

The value

$$\frac{1}{2}\,(G_x^{\mathrm{I}}+G_x^{\mathrm{II}}).x+\frac{1}{2}\,(G_y^{\mathrm{I}}+G_y^{\mathrm{II}}).y=\delta\Delta g_E$$

is essentially the variation of Δg between the points P_{I} and P_{II} computed from the gradient values in accordance with Eötvös computation method.

At last the correction term:

$$D=\left[\left(\frac{\partial G_x^{\mathrm{II}}}{\partial x}-\frac{\partial G_x^{\mathrm{I}}}{\partial x}\right).x^2+2\left(\frac{\partial G_x^{\mathrm{II}}}{\partial y}-\frac{\partial G_x^{\mathrm{I}}}{\partial y}\right).xy+\ldots\right]$$

contains only second order terms and the values are attained by the aid of torsion balance sections obtained from the measurements made on the area.

Using the above abbreviations, the average value of vertical gradient, referring to the two observation sites, may be given by the following formula:

$$\frac{G_z^{\mathrm{I}}+G_z^{\mathrm{II}}}{2}=\frac{1}{z}\left[\delta\Delta g-\delta\Delta g_E+\frac{1}{12}\cdot D\right]$$

The validity of this formula is demonstrated on the multiple used two-dimensional structure, characteristic for anomalies in figures 6 a,b,c; 7 a,b,c; 8 a,b,c resp. 9 a,b,c. In these diagrams the thick continous lines represent the theoretical average values of the vertical gradients belonging to two neighbouring points with an elevation difference of 100 m. The thin continous lines represent the same values, calculated from the above formula, but neglecting the value of the correction term D, while the dotted lines give the values calculated by using this correction term. Where this dotted line is not visible, it coincides with the theoretical value.

The figures 6 and 7 represent the differences between the theoretical and computed average-values of vertical gradient by a station distance of 200 m, the figures 8 and 9 by a distance of 400 m. The number written beside the sections give the distance between the top of the structure and the lower of the two observation level.

The effects are calculated by the supposition that the Δg-values may be determined to 0,01 milligal, while the gradient-values to an accuracy of 1 E.

As the term D is mostly negligible, omitting it, the result can be summarized as follows:

The average value of vertical gradients referring to two in different elevation lying observation stations may be obtained by dividing the deviation resulting from the comparison of Δg-differences observed by the gravity meter, with the Δg-differences calculated from the gradient values in accordance with Eötvös' method ,by the elevation difference of two observation stations.

The above formula in the case of $z=0$, i. e. when the two points of observation are in the same level, becomes

$$\delta\Delta g=\delta\Delta g_E-\frac{1}{12}\left[\left(\frac{\partial G_x^{\mathrm{II}}}{\partial x}-\frac{\partial G_x^{\mathrm{I}}}{\partial x}\right)x^2+2\left(\frac{\partial G_x^{\mathrm{II}}}{\partial y}-\frac{\partial G_x^{\mathrm{I}}}{\partial y}\right)xy+\left(\frac{\partial G_y^{\mathrm{II}}}{\partial y}-\frac{\partial G_y^{\mathrm{I}}}{\partial y}\right)y^2\right]$$

This formula shows that in an area, *where great variations of gradients are encountered, the calculation of isogams from the gradient values may be executed only with the aid of third order derivatives,* Therefore in a flat area the gravity meter data are more reliable concerning the isogams values, than those calculated on the ground of torsion balance measurements.

But the results obtained enables us to get the true values of vertical gradients instead of the average values. Namely from the observations made in three points in different levels the following values can be calculated:

$$\frac{1}{2}(G_z^{I}+G_z^{II});\ \frac{1}{2}(G_z^{II}+G_z^{III}) \text{ and } \frac{1}{2}(G_z^{I}+G_z^{III})$$

Using these values

$$G_z^{I}=\frac{1}{2}(G_z^{I}+G_z^{II})+\frac{1}{2}(G_z^{II}+G_z^{III})-\frac{1}{2}(G_z^{II}+G_z^{III})$$

and likewise may be obtained also the values G_z^{II} and G_z^{III}.

Having only one observation in different level, then by the aid of the above method the value $\frac{1}{2}(G_z^{I}+G_z^{II})$ is obtained. But comparing the formula of G_z -variations

$$G_z^{II}-G_z^{I}=\frac{1}{2}\left(\frac{\partial G_x^{I}}{\partial z}+\frac{\partial G_x^{II}}{\partial z}\right).x+\frac{1}{2}\left(\frac{\partial G_y^{I}}{\partial z}+\frac{\partial G_y^{II}}{\partial z}\right).y+\frac{1}{2}\left(\frac{\partial G_z^{I}}{\partial z}+\frac{\partial G_z^{II}}{\partial z}\right).z$$

with the former quantity, then

$$G_z^{II}=\frac{1}{2}(G_z^{I}+G_z^{II})+\frac{1}{2}(G_z^{I}-G_z^{II})$$

and in similar way the other values .

To obtain reliable data for the vertical gradient it is necessary to work with an accuracy of 0,01 milligal in gravity meter data and *1E* in torsion balance data .The accuracy of vertical gradient is greater, than that of the curvature.

5. At last an application of the obtained results is given to depth estimation concerning vertical and cylindric masses (see fig. 10.).

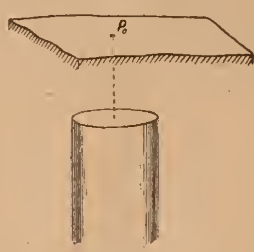


Fig. 10.

In the case of a vertical cylindric body (a salt dom is good approximation for such a body) the gravity effect on the surface, observed in axis is

$$\frac{\partial U}{\partial z}=f\Delta\sigma\int_o^R\int_h^\infty\int_o^{2\pi}\frac{\varrho\zeta}{(\varrho^2+\zeta^2)^{3/2}}\,d\varphi d\zeta d\varrho=2\pi f\Delta\sigma\,(\sqrt{R^2+h^2}-h).$$

$$\frac{\partial^2 U}{\partial z^2}=f\Delta\sigma\int_o^R\int_h^\infty\int_o^{2\pi}\left[\frac{\varrho}{(\varrho^2+\zeta^2)^{3/2}}-\frac{3\zeta^2\varrho}{(\varrho^2+\zeta^2)^{5/2}}\right]d\varphi d\zeta d\varrho=2\pi f\Delta\sigma\,\frac{\sqrt{R^2+h^2}-h}{\sqrt{R^2+h^2}}$$

Forming the quotient of these two values, the following formula is received:

$$\eta = \sqrt{R^2 + h^2} \qquad \text{where} \qquad \eta = \frac{\frac{\partial U}{\partial z}}{\frac{\partial^2 U}{\partial z^2}} \qquad \text{and thus} \qquad h = \sqrt{\eta^2 - R^2};$$

a form, where the density, which makes the most difficulties, is absent. From the derivatives and the knowledge of R, the value of h can be calculated, independent of the density differences between the two masses. The value of R may be obtained generally from the gradient values, because these indicate the edge of the structure with a good approximation.

UJ BALANIDÁK A HAZAI HARMADKORBÓL

Irta: KOLOSVÁRY GÁBOR

7 ábrával

Hazai fosszilis faunánkból eddig a korallokban lakó kacslábú rákok közül csak a VADÁSZ ELEMÉR által először megtalált „*Pyrgoma multicostatum* SEGUENZA" volt ismeretes. (Lásd utalást: „Földt. Közl." Centennáris kötet p. 102—112.) E faj helyes neve: *Creusia Rangii* (DESMOUL.), synonimái: *Pyrgoma Rangii* DES MOULINS, *Pyrgoma multicostatum* SEGUENZA, *Pyrgoma costatum* GORJANOVIC-KRAMBERGER és *Creusia Funchi* PROCHASKA (lásd utalást: T. H. WITHERS: The phylogeny of the Cirripeds in „Ann. Mag. Nat. Hist." 10, Vol. IV. 1929 p 559—566.). Ez a faj azonban nem azonos az ABEL által leírt *Paracreusia Trolli* nevű fajjal (lásd: O. ABEL: Vorzeitliche Lebensspuren, Verl. Fischer, Jena, 1935.) — A *Creusia Rangii* (DESMOUL.) eddig előkerült francia, olasz, osztrák, horvát, erdélyi és magyarországi miocén rétegekből *Orbicella (Heliastraea)* és *Isastraea* korallokban.

A Creusiák abban különböznek a Pyrgomáktól, hogy héjuk négy lemezből áll, a Pyrgómáké ellenben egységes lemezzé olvad össze. Az összetévesztés ennek a bélyegnek félreismeréséből ered. — Élő *Creusia*-faj a: *Creusia spinulosa* LEACH 1824, mely 13 formára bomlik, s valamennyi ma is kizárólagosan korallokban él.

VADÁSZ ELEMÉR felfedezése óta a következő új adatokról számolhatok be:

a) *Creusia spinulosa forma cladangiae n. f. foss.* Sámsonháza, a Halastó-hegy nyugati öble. Miocénkorú alaprétegek andezittufa felett. Osztrigás agyagpadból gyüjtötte LEGÁNYI FERENC. A példányok egy *Cladangia conferta* REUSS nevü korall törzstöredékéből valók. Leírása az angolnyelvű szövegben (1. ábra).

b) *Creusia spinulosa forma praespinulosa n. f. foss.* Pécsbudafa, temető alatti régi mély út rétegeiből *Ostrea gingensis*-padból. Középső miocén torton-emelet. Lajtamészkőben ostreás kavicspad. A példányok korallon kívül kavicsos homokkőben vannak, a kőzet vasas oldattal és erős kalcitosodással. Két példány az *Ostrea gingensis* héjára megtelepült (lásd 5. ábrát). A kőzetben különben még *Pholas* s egyéb molluszkanyomok is vannak.